130. A Note on Semi-prime Modules. I

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Feller and Swokowski [1, 2] have generalized Goldie's works on prime and semi-prime rings [3, 4] to modules.

It is the aim of the present note to investigate these modules and semi-prime Goldie rings. This note lays a result concerning the dimensions of semi-prime modules and of semi-prime Goldie rings. We can prove that the set of the subisomorphism classes of basic submodules of a semi-prime R-module M corresponds one-to-one onto the set of the minimal annihilator ideals of the semi-prime Goldie ring R (see Theorem 7) under no maximum conditions for right complements and for right annihilators of M. The relationship between prime and semi-prime modules is also studied, and Theorem 8 shows that clM_i is a prime R_i-module, where clM_i is a homogeneous component of M, and R_i is the minimal annihilator ideal of R which corresponds to clM_i.

Throughout this paper, R will denote a right Goldie ring; that is

(a) R satisfies the maximum condition for right complements;

(b) R satisfies the maximum condition for right annihilators.

All R-modules will mean faithful right R-modules. If M and N are R-modules, then M is an essential extension of N if $N \subseteq M$ and $N \cap L \neq 0$ for every non-zero submodule L of M. In this case, we call N a large submodule of M. We shall also speak of large right ideals of R by considering R as a right module over itself. Let M be an R-module and let X and Y be subsets of M and R respectively, then the annihilators are defined as $X_r = \{a \in R \mid xa = 0 \text{ for all } x \in X\}$ and $Y_l = \{m \in M \mid my = 0 \text{ for all } y \in Y\}$. The closure clN of a submodule N of M is defined by $clN = \{m \in M \mid mL \subseteq N : L \text{ a large right ideals of } R\}$. If $clN = N$, then N is said to be closed. If R is a semi-prime Goldie ring, then according to Theorem 5 in [4], a right ideal of R is large if and only if it contains a regular element. Hence, in this case, $clN = \{m \in M \mid mc \in N : c \text{ a regular element of } R\}$. The singular submodule $M^{\blacktriangle}$ of M is defined as $cl0$. Let A be a right ideal of R. Then the singular submodule of A-module M is denoted by $(M_A)^{\blacktriangle}$. As in [2], an R-module M is said to be semi-prime if the prime radical $P(M)$[1)]

1) Cf. [2, p. 825].