182. *On the Type of an Associative H-space of Rank Three*

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1. Introduction. A topological space X with a continuous multiplication with unit is called an H-space. If this multiplication is associative, X is called an associative H-space. Suppose that X is an associative H-space and that the integral cohomology of X is finitely generated. Then it follows from the classical Hopf's theorem that the rational cohomology of X is an exterior algebra on a finite number of odd dimensional generators. The number of such generators is called the rank of X. The dimensions in which the generators occur is called the type of X. L. Smith determined all the possible types of associative H-spaces of rank 2 [6].

In this paper, we apply L. Smith's method to an associative H-space of rank 3 and determine the types of such a space.

Theorem. *Let X be an arcwise connected H-space of rank* 3 *with $H_*(X;Z)$ finitely generated as an abelian group. Then the type of X is either* (1, 1, 1), (1, 1, 3), (1, 3, 3), (1, 3, 5), (1, 3, 7), (1, 3, 11), (3, 3, 3), (3, 3, 5), (3, 3, 7), (3, 3, 11), (3, 5, 7), (3, 7, 11), (3, 5, 5), (3, 5, 11), (3, 7, 7), *or* (3, 11, 11).

Examples of H-spaces having the types from (1, 1, 1) to (3, 7, 11) are given by $S^1\times S^1\times S^1$, $S^1\times S^1\times S^3$, $S^1\times S^3\times S^3$, $S^1\times SU(3)$, $S^1\times SP(2)$, $S^1\times G_2$, $S^3\times S^3\times S^3$, $S^3\times SU(3)$, $S^3\times SP(2)$, $S^3\times G_2$, $SU(4)$, and $SP(3)$ respectively.

The author does not know whether the remaining types are realized or not.[1] I wish to express my hearty thanks to L. Smith for suggesting this problem and giving me many helpful advices, and to Professors K. Morita and R. Nakagawa for their criticism and encouragement.

2. Some results on unstable polyalgebras. A polynomial algebra R over the mod p Steenrod algebra A_p(p : prime) is called an unstable polyalgebra over A_p, if it is an algebra that is left A_p-module satisfying

1) After the manuscript had been submitted, L. Smith suggested to me that it was possible to show that an H-space of type (3, 5, 5), (3, 5, 11), (3, 7, 7) or (3, 11, 11) did not exist. In fact, the non-existence of H-spaces with the types (3, 5, 11) and (3, 11, 11) is proved by using the Steenrod operation P^2.