181. On Nuclear Spaces with Fundamental System of Bounded Sets. II

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A locally convex vector space with a countable fundamental system of bounded sets has already been developed in several bibliographies. Barrelled spaces and quasi-barrelled spaces with a countable fundamental system of compact sets has been studied by J. Dieudonné [2] and by M. Mahowald and G. Gould [7] respectively.

We considered, the open mapring and closed graph theorems on a nuclear dualmetric space in the previous paper [4].

Let E be a normed space then E is a nuclear space if and only if it is finite dimentional. It is also known that a normed space can only be a Montel (i.e., barrelled and perfect) space if it is finite dimensional. In this paper, we prove a nuclear dualmetric space which is quasi-complete is Montel space, and using this result, we consider analogous theorem to M. Mahowald and G. Gould [7], in nuclear space.

For nuclear spaces and its related notion, see A. Pietsch [8] and S. Funakosi [4]. Most of the definitions and notations of the locally convex vector spaces are taken from N. Bourbaki [1] and T. Husain [5].

Definition. *Let E be a locally convex space and E' its dual.*

(1) *If only all countable strong bounded subset of E' are equicontinuous, then E is called the σ-quasi-barrelled.*

(2) *Let E be a σ-quasi-barrelled space, if there exists a countable fundamental system of bounded subset in E, then E is called the dualmetric space.*

The following Lemma is well known.

Lemma 1. *A metric or dualmetric locally convex vector space E is nuclear if and only if its dualnuclear.*

The proof is given in A. Pietsch [8].

Proposition 1. *Each nuclear dualmetric space E is a quasi-barrelled.*

Proof. By Lemma 1, the strong dual E'^{β} is nuclear, so an arbitrary bounded subset of E'^{β} is separable (see, the proof of Theorem 4, (a) in S. Funakosi [4]). Denote by B strong bounded subset of E', then $B \subseteq \overline{\{a_n ; a_n \in B\}}$. On the other hand, since E is dualmetric it is a