174. An Algebraic Formulation of K-N Propositional Calculus. IV

By Shôtarô TANAKA

(Comm. by Kinjirô KUNUGI, M. J. A., Oct. 12, 1968)

In his paper [1], K. Iséki has defined the NK-algebra. For the details of the NK-algebra, see [1]. The conditions of the NK-algebra are as follows:

a) $\sim(p*p)*p=0$,

b) $\sim p*(q*p)=0$,

c) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$,

d) Let α, β be expressions in this system, then $\sim\sim\beta*\sim\alpha=0$ and $\alpha=0$ imply $\beta=0$.

In my paper [2], I showed that the NK-algebra is characterized by the following conditions:

a) $\sim(p*p)*p=0$,

b′) $\sim q*(q*p)=0$,

c) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$,

d) Let α, β be expressions in this system, then $\sim\sim\beta*\sim\alpha=0$ and $\alpha=0$ imply $\beta=0$.

1) $\sim(p*p)*p=0$.

2) $p*(q*\sim p)=0$.

3) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$.

4) $\sim\sim\beta*\sim\alpha=0$ and $\alpha=0$ imply $\beta=0$, where α, β, are expressions in this system. We shall show that 1)–4) imply b′).

In 3), put $p=\beta$, $q=\alpha$, $r=\gamma$, then by 4) we have

A) $\sim\alpha*\beta=0$ implies $\sim\sim(\beta*\gamma)*\sim(\gamma*\alpha)=0$.

Then we have

B) $\sim\alpha*\beta=0$ and $\gamma*\alpha=0$ imply $\beta*\gamma=0$.

C) $\sim\alpha*\beta=0$ and $\sim\gamma*\alpha=0$ imply $\beta*\sim\gamma=0$.

In B), put $\alpha=\sim p*\sim p$, $\beta=\sim p$, $\gamma=p$, then by 1) and by 2) we have

5) $\sim p*p=0$.

In 3), put $q=p$, then $\sim\sim(\sim\sim(p*r)*\sim(r*p))*\sim(\sim p*p)=0$. By 5) we have

6) $\sim\sim(p*r)*\sim(r*p)=0$.

In 6), put $p=\alpha$, $r=\beta$, then $\sim\sim(\alpha*\beta)*\sim(\beta*\alpha)=0$. Hence by 4) we have

D) $\beta*\alpha=0$ implies $\alpha*\beta=0$.