167. On the Absolute Convergence of Fourier Series

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1. The following theorems are due to Izumi [2]:

Theorem A. *Let* $f(t) \sim \sum_{1}^{\infty} a_n \cos nt$. *If*

(i) $\int_0^\pi \log \frac{2\pi}{t} |df(t)| < \infty$ *and* (ii) $\{n^\delta \Delta(na_n)\} \in BV$

for some $\delta > 0$, *then* $\sum |a_n| < \infty$.

Theorem B. *Let* $g(t) \sim \sum_{1}^{\infty} b_n \sin nt$. *If*

(i)* $\int_0^\pi \log \frac{2\pi}{t} |dg(t)| < \infty$ *and* (ii)* $\{n^\delta \Delta(nb_n)\} \in BV$

for some $\delta > 0$, *then* $\sum |b_n| < \infty$.

Theorem C. *Let* $f(t) \sim \sum_{1}^{\infty} a_n \cos nt$. *If*

(i)′ $f(t) \in BV(0, \pi)$ *and* (ii)′ $\{n^\delta \Delta(na_n)\} \in BV$
for some $\delta > 0$, *then* $\sum |a_n| / \log n < \infty$.

Theorem D. *Let* $f(t) \sim \sum_{1}^{\infty} a_n \cos nt$ *and let* $\alpha > \beta + 2$ *and* $\beta > 0$. *If*

(i)″ $\int_0^\pi t^{-1/\beta} |df(t)|$ *and* (ii)″ $\{(\log n)^\alpha \Delta(na_n)\} \in BV$,

then $\sum |a_n| < \infty$.

In this note the following theorems will be established which are generalizations of the results mentioned above:

Theorem 1. *Let* $f(t) \sim \sum_{1}^{\infty} a_n \cos nt$. *If*

$$\int_0^\pi \log \frac{k}{t} |df(t)| < \infty \tag{1.1}$$

and

$$\left\{ \frac{1}{e^{n^\alpha}} \sum_{1}^{n} e^{v^\alpha} a_v \right\} \in BV, \qquad 0 < \alpha < 1, \tag{1.2}$$

then $\sum |a_n| < \infty$.

Theorem 2. *Let* $g(t) \sim \sum_{1}^{\infty} b_n \sin nt$ *with* $g(+0) = 0$. *If*

$$\int_0^\pi \log \frac{k}{t} |dg(t)| < \infty \tag{1.3}$$

and (1.2) *holds, then* $\sum |b_n| < \infty$.