213. *On Extensions of Mappings into n-Cubes*

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1. Introduction. The purpose of this note is to give a generalization of the results of M. K. Fort, Jr. [1] to the case of arbitrary metric spaces.

Let X be a metric space and $\dim X$ the covering dimension of X. We denote by I^n the closed unit cube in Euclidean n-space, where $n>0$. If A is a subset of X and f is a mapping whose domain contains A, *f is of type k on A* if and only if $\dim (f^{-1}(y)\cap A)\leqq k$ for all y in the range of f, where $k\geqq -1$. In the following, a mapping means always a continuous transformation.

Let us assume that A is a closed subset of X, $\dim(X-A)=m\geqq n$ and f is a mapping of A into I^n. It will be shown that f can be extended to a mapping φ of X into I^n such that φ is of type $m-n$ on $X-A$. Under the assumption of separability for X, this theorem was proved by A. L. Gropen [2] and essentially by M. K. Fort, Jr. [1]. If f is, in addition, of type $m-n$ on A, it will also be shown that the mapping φ, whose existence is asserted above, is of type $m-n$ on X. These results will be established in §3.

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2. Auxiliary lemmas. We employ the terminology of M. K. Fort, Jr. [1]. A finite collection Σ of subsets of a metric space has Property D if and only if there exists $\varepsilon>0$ such that any set which contains at least one point from each member of Σ has diameter greater then ε. If A is a closed subset of a metric space X and f is a mapping into I^n whose domain contains A, we let $C_n(f\mid A)$ be the space of mappings g of X into I^n for which $g\mid A=f\mid A$ metrized by the uniform metric. By the Tietze extension theorem, $C_n(f\mid A)$ is non-empty and is a complete metric space.

The following Lemma 1 was proved by M. K. Fort, Jr. In his paper [1], it was assumed that X is a separable metric space, but by virtue of [5, p. 49] the separability of X is not necessary.

Lemma 1. *If A is a closed subset of a metric space X, f is a mapping of A into I^n and $F_0, \cdots, F_n$ are mutually exclusive subsets of $X-A$ which are closed in X and each of dimension less than n, then*