210. *Semifield Valued Functionals on Linear Spaces*

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An ordered (real) linear space E is defined as a linear space with an order relation satisfying the following conditions:

1) $x \leqslant y$ implies $x+z \leqslant y+z$,

2) $x \leqslant y$, $0 \leqslant \alpha$ imply $\alpha x \leqslant \alpha y$

for every x, y in E. Then $K=\{x \mid x \geqslant 0\}$ is a convex cone, i.e., K has the following properties:

3) $K+K \subset K$,

4) $\alpha K \subset K$ for every positive real number α,

5) $K \cap (-K)=$ the zero element of E.

As well known, for a real linear space, there is a one-to-one correspondence between all order relations 1), 2) and all convex sets with properties 3)–5). For details of ordered linear spaces, [1], [3]–[5].

In this Note, we shall consider semifield valued functionals on E. Unless the contrary is mentioned, functionals mean semifield valued functionals.

We shall prove a theorem which is a generalization of our result [2]. In our discussion, we follow the techniques by M. Cotlar and R. Cignoli [1].

Theorem 1. *Let E be an ordered linear space, K its associated cone, and G a linear subspace of E. Let $p(x)$ be a sublinear functional on E, $f(x)$ a linear functional on F satisfying*

$$f(y) \ll p(y+z) \quad \text{for all} \quad y \in G,\ z \in K. \tag{1}$$

Then there is a linear extension $F(x)$ on E of $f(x)$ such that

$$F(x) \ll p(x+z) \quad \text{for all} \quad x \in E,\ z \in K.$$

The notion of semifields was introduced by M. Antonovski, V. Boltjanski and T. Sarymsakov. For the notations used, see my reviews of their books, Zentralblatt für Mathematik, **142**, pp. 209–211 (1968).

Proof. Let $E-G \neq \emptyset$, and we take an element $x_0 \in E-G$. Then each element x of the linear space $G_1=(G, x_0)$ generated by G and x_0 is uniquely represented in the form of $x=x' \pm \alpha x_0 (x' \in G, \alpha>0)$. We shall extend the linear functional $f(x)$ on G_1. Consequently, by the transfinite method or the Zorn lemma, we have a linear functional $F(x)$ satisfying the conditions mentioned in Theorem 1.

Let $y_1, y_2 \in G$, $z_1, z_2 \in K$, then by (1) we have