209. *A Simple Characterization of Boolean Rings*

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G. R. Blakley, S. Ôhashi, and the present author give some new axioms for commutative rings (see [1]-[4]). In this Note, we shall give a new axiom system of Boolean rings.

Let $<R, +, \cdot, -, 0, 1>$ be an algebraic system, where R is a non-empty, 0 and 1 are elements of R, $+$, and . are binary operations on R, and $-$ is a unary operation on R.

Then we have the following

Theorem. *$<R, +, \cdot, -, 0, 1>$ is a Boolean ring, if it satisfies the following conditions:*

1) $r=r+0,$

2) $r1=1r=r,$

3) $((-r)+r)a=0,$

4) $((ay+bx)+cr)r=b(rx)+(a(yr)+cr).$

In our discussion, we do not use the multiplication symbol dot. Therefore ab means $a\cdot b$.

The proof of Theorem follows from the following several steps.

5) $(-r)+r=0.$

$0=((-r)+r)1$ {3}

$=(-r)+r.$ {2}

6) $0a=0.$

$0a=((-r)+r)a=0.$ {5, 3}

7) $a+b=b+a.$

$a+b=((a1+b1)+01)1$ {1, 2, 6}

$=b(11)+(a(11)+01)$ {4}

$=b+a.$ {1, 2, 6}

8) $(ay)r=a(yr).$

$(ay)r=((ay+0x)+0r)r$ {1, 6}

$=0(rx)+(a(yr)+0r)$ {4}

$=a(yr).$ {1, 6, 7}

9) $(a+b)+c=a+(b+c).$

$(a+b)+c=(b+a)+c$ {7}

$=((b1+a1)+c1)1$ {2}

$=a(11)+(b(11)+c1)$ {4}

$=a+(b+c).$ {2}

10) $(a+b)r=ar+br.$