207. *On Axiom Systems of Commutative Rings*

By Sakiko ÔHASHI

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Recently G. R. Blakley gives an interesting axiom system of commutative rings (see G. R. Blakley [1]).

In this short Note, we give some new axioms of commutative rings and semirings that the addition and multiplication are commutative.

Theorem 1. *A set with two nullary operations,* 0 *and* 1, *with one unary operation,* $-$, *and with two binary operations,* $+$ *and juxtaposition such that*

1) $r+0=r,$

2) $r1=r,$

3) $((-r)+r)a=0,$

4) $((ay+bx)+cz)r=b(xr)+(a(yr)+z(cr))$

for every a, b, c, r, x, y, z, *is a commutative ring with unit element.*

Remark. It is obvious that every commutative ring (with unit element) satisfies 1)—4).

Proof. The proof is divided into several steps.

5)
$$\begin{aligned} &(-r)+r \\ &=((-r)+r)1 && \{2\} \\ &=0. && \{3\} \end{aligned}$$

6)
$$\begin{aligned} &0a \\ &=((-0)+0)a && \{5\} \\ &=0. && \{3\} \end{aligned}$$

7)
$$\begin{aligned} &a+b \\ &=((a1+b1)+00)1 && \{2, 6\} \\ &=b(11)+(a(11)+0(01)) && \{4\} \\ &=b+a. && \{2, 6, 1\} \end{aligned}$$

8)
$$\begin{aligned} &cz \\ &=((00+00)+cz)1 && \{1, 7, 6, 2\} \\ &=0(01)+(0(01)+z(c1)) && \{4\} \\ &=zc. && \{1, 7, 2\} \end{aligned}$$

9)
$$\begin{aligned} &(b+a)+c \\ &=(a+b)+c && \{7\} \\ &=((a1+b1)+c1)1 && \{2\} \\ &=b(11)+(a(11)+1(c1)) && \{4\} \\ &=b+(a+c). && \{2\} \end{aligned}$$