204. On the Product of M-Spaces. II

By Tadashi ISHII, Mitsuru TSUDA, and Shin-ichi KUNUGI
Utsunomiya University

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1. This is the continuation of our previous paper [1].*) The purpose of this paper is to prove the following theorems which are related to the product of M-spaces and to the countable product of the spaces belonging to $\mathfrak{C}$.

Theorem 1.1. *If X belongs to $\mathfrak{C}$, then the product $X \times Y$ is an M-space for any M-space Y.*

Corollary 1.2. *If X is an M-space which satisfies one of the following conditions, then the product $X \times Y$ is also an M-space for any M-space Y.*

(a) *X satisfies the first axiom of countability.*

(b) *X is locally compact.*

(c) *X is paracompact.*

Since an M-space X which satisfies one of conditions (a), (b), and (c) belongs to $\mathfrak{C}$ by [1, Theorem 2.2], this corollary is a direct consequence of Theorem 1.1.

Theorem 1.3. *If $X_n, n=1, 2, \cdots$, are the spaces belonging to $\mathfrak{C}$, then the product $\prod_{n=1}^{\infty} X_n$ also belongs to $\mathfrak{C}$.*

Corollary 1.4. *If $X_n, n=1, 2, \cdots$, are M-spaces each of which satisfies the first axiom of countability, then the product $\prod_{n=1}^{\infty} X_n$ is also an M-space satisfying the first axiom of countability.*

If each space X_n satisfies the first axiom of countability, then the product $\prod_{n=1}^{\infty} X_n$ satisfies the first axiom of countability, too. Hence this corollary follows from Theorem 1.3 directly.

If each space X_n is a paracompact M-space, then the product $\prod_{n=1}^{\infty} X_n$ is also a paracompact M-space (cf. K. Morita [3, Theorem 6.4]). However for locally compact M-spaces X_n, the product $\prod_{n=1}^{\infty} X_n$ is not locally compact in general. For example, let $X_n, n=1, 2, \cdots$, be the spaces of real numbers with the usual topology. Then the product $\prod_{n=1}^{\infty} X_n$

*) All spaces are assumed to be Hausdorff.