# 203. On the Product of M-Spaces. I

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**1. Introduction.** In the present paper all spaces are assumed to be Hausdorff. In his previous paper [2], K. Morita has introduced the notion of $M$-spaces. A space $X$ is called an $M$-space if there exists a normal sequence $\{\mathfrak{U}_i \mid i=1, 2, \cdots\}$ of open coverings of $X$ satisfying the condition (M) below:

(M) If $\{K_i\}$ is a sequence of non-empty subsets of $X$ such that $K_{i+1} \subset K_i$, $K_i \subset \mathrm{St}(x_0, \mathfrak{U}_i)$ for each $i$ and for some fixed point $x_0$ of $X$, then $\cap \bar{K}_i \neq \phi$.

As is easily verified, Condition (M) is equivalent to the condition ($M_0^*$) below:

($M_0$) If $\{x_i\}$ is a sequence of points of $X$ such that $x_i \in \mathrm{St}(x_0, \mathfrak{U}_i)$ for each $i$ and for some fixed point $x_0$ of $X$, then $\{x_i\}$ has an accumulation point.

Hereafter we use Condition ($M_0$) in place of Condition (M).

As for the product $X \times Y$ of two $M$-spaces $X$ and $Y$, it seems to be unknown whether $X \times Y$ is also an $M$-space or not. We can give an affirmative answer for this problem in the following cases:

(a) $X$ satisfies the first axiom of countability.

(b) $X$ is locally compact.

(c) $X$ is paracompact.

The purpose of our papers I and II is to introduce the notion of the spaces belonging to the class $\mathfrak{C}$ and to prove a more general theorem (cf. Theorem 1.1 in II) as follows: If a space $X$ belongs to the class $\mathfrak{C}$, then the product $X \times Y$ is also an $M$-space for any $M$-space $Y$. We denote by $\mathfrak{C}$ the class of all spaces $X$ such that there exists a normal sequence $\{\mathfrak{U}_i\}$ of open coverings of $X$ satisfying the condition (*) below:

(*) If $\{x_i\}$ is a sequence of points of $X$ such that $x_i \in \mathrm{St}(x_0, \mathfrak{U}_i)$ for each $i$ and for some fixed point $x_0$ of $X$, then there exist a subsequence $\{x_{i(n)} \mid n=1, 2, \cdots\}$ of $\{x_i\}$ which has the compact closure.

The class $\mathfrak{C}$ contains all $M$-spaces satisfying one of conditions (a), (b), and (c), and further the spaces belonging to $\mathfrak{C}$ have the following properties.

(i) If $f: X \to Y$ is a quasi-perfect map (i.e., a continuous closed