202. *An Asymptotic Property of Gaussian Stationary Processes*

By Hisao WATANABE
Kyûsyû University

(Comm. by Kunihiko KODAIRA, M. J. A., Nov. 12, 1968)

Let $X=\{x(t), -\infty<t<\infty\}$ be a real separable stochastically continuous Gaussian stationary process defined on a probability measure space $(\Omega, \mathscr{B}, P)$. We assume that $E(x(t))=0$ and $E(x^2(t))=1$. We put $r(t)=E(x(t)x(0))$ and $\sigma^2(h)=E((x(t+h)-x(t))^2)$.

If the sample functions are almost certainly everywhere continuous, for every fixed $T>0$, the quantity

$$\eta(T)=\max_{0\leqq t\leqq T} x(t)$$

will have a definite meaning. In this note, we announce some results on the asymptotic behaviour of the processes $\{x(t), -\infty<t<\infty\}$ and $\{\eta(t), -\infty<t<\infty\}$.

We introduce the following conditions:

A, 1) There are constants C_1, δ_1 such that

$$\sigma^2(h)\leqq C_1 h^\alpha$$

for all h in $(0, \delta_1)$ for some α with $0<\alpha\leqq 2$.

A, 1′) There are constants C_2, δ_2 such that

$$C_2 h^\alpha \leqq \sigma^2(h)$$

for all h in $(0, \delta_2)$ for some α with $0<\alpha\leqq 2$.

A, 2) $\operatorname*{Lim\,sup}_{t\to\infty} r(t)\log t\leqq 0$.

Theorem 1. *Suppose that the condition* A, 1) *is satisfied and that* $\sigma^2(h)$ *is monotone non-decreasing in* $(0, \delta_1)$. *Let* $\varphi(t)$ *be a monotone non-decreasing continuous function for large* t's. *If*

$$\int^\infty \varphi(t)^{\frac{2}{\alpha}-1} \exp\left(-\frac{1}{2}\varphi^2(t)\right)dt<+\infty,$$

then we have

$$P(\text{there is a } t_0(\omega) \text{ such that } x(t)\leqq\varphi(t) \text{ for all } t\geqq t_0)=1$$

or equivalently

$$P(\text{there is a } T_0(\omega) \text{ such that } \eta(T)\leqq\varphi(T) \text{ for all } T\geqq T_0)=1.$$

Theorem 2. *Suppose that conditions* A, 1′) *and* A, 2) *are satisfied and that* $\sigma^2(h)$ *is monotone non-decreasing function in* $(0, \sigma_2)$. *Let* $\varphi(t)$ *be a monotone non-decreasing function for large* t's.

If

$$\int^\infty \varphi(t)^{\frac{2}{\alpha}-1} \exp\left(-\frac{1}{2}\varphi^2(t)\right)dt=\infty,$$