201. On Numbers Expressible as a Weighted Sum of Powers

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1. In a recent paper [3] we proved

Theorem 1. *There is n_0 such that for every $n \geqq n_0$ there are positive integers x and y satisfying*

$$n < x^f + y^h < n + cn^p$$

where f and h are any integers such that $f \geqq h \geqq 2$,

$$c = hf^{1-(1/h)} \text{ and } p = \left(1 - \frac{1}{f}\right)\left(1 - \frac{1}{h}\right).$$

Mordell [4] has recently proved

Theorem 2. *There are non-negative integers $x_1, \cdots, x_k$ satisfying*

$$n \leqq a_1 x_1^{h_1} + \cdots + a_k x_k^{h_k} < n + cn^p + O(n^{p(h_k-2)/(h_k-1)})$$

where $a_1, \cdots, a_k \geqq 1$, $1 < h_1 \leqq h_2 \leqq \cdots \leqq h_k$,

$$c = (a_1^{1/h_1} h_1)(a_2^{1/h_2} h_2)^{1-(1/h_1)}(a_3^{1/h_3} h_3)^{(1-(1/h_1))(1-(1/h_2))} \cdots (a_k^{1/h_k} h_k)^{(1-(1/h_1))\cdots(1-(1/h_{k-1}))}$$

and

$$p = \left(1 - \frac{1}{h_1}\right) \cdots \left(1 - \frac{1}{h_k}\right).$$

Theorem 1 generalizes some results previously obtained by Bambah and Chowla [1], Uchiyama [5] and the author [2] while Theorem 2 deals with a problem more general than those discussed in [1], [5], [2] and [3].

In this note we prove the following generalization of Theorem 1 and refinement of Theorem 2:

Theorem 3. *There is n_0 such that for every real $n \geqq n_0$ there are positive integers $x_1, \cdots, x_k$ satisfying*

$$n < a_1 x_1^{h_1} + \cdots + a_k x_k^{h_k} < n + cn^p$$

where $a_1, \cdots, a_k$ are real and >0, $h_1, \cdots, h_k$ are real and >1, $k>1$, c and p are as in Theorem 2 and

$$a_1 h_1^{h_1} \leqq a_2 h_2^{h_2} \leqq \cdots \leqq a_k h_k^{h_k}.$$

In what follows we write $[t]$ for the greatest integer $\leqq t$.

2. We first prove the following generalization of Theorem 4A of [2]:

Theorem 4. *Let a and $b>0$, f and $h>1$,*

$$N = N(n) = a\{(n/a)^{1/f} + 1\}^f - n + b$$

and