200. An Extension of Wild's Sum for Solving Certain Non-linear Equation of Measures

By Shigeru TANAKA
Department of Mathematics, Osaka University, Osaka

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1. Let S be a compact space with the second countability axiom. Let $\mathfrak{M}$ be the set of all signed measures on the topological Borel field of S with finite total variation, and let $\mathfrak{M}_s$ (resp. $\mathfrak{M}_p$) be the subset of $\mathfrak{M}$ of all substochastic (resp. probability) measures. In $\mathfrak{M}$, we introduce the topology of weak convergence. Consider a non-linear equation:

$$(1) \qquad \frac{du(t)}{dt} = B[u(t)] - u(t), \qquad u(0+) = f,^{1)}$$

where the initial value f and the solution $u(t)$ are in $\mathfrak{M}_s$ and $B[u]$ is given by the formula:

$$(2) \qquad B[u] = \sum_{n=1}^{\infty} a_n B_n[u, \cdots, u],$$

for given $\{a_n\}_{n=1}^{\infty}$ and $\{B_n\}_{n=1}^{\infty}$ such that i) a_n is a non-negative real number, $a_1 < 1$ and $\sum_{n=1}^{\infty} a_n = 1$, ii) B_n is a mapping from $\mathfrak{M}^n$ to $\mathfrak{M}$, multi-linear, continuous and maps $\mathfrak{M}_p^n$ into $\mathfrak{M}_p$, for each $n \geqq 1$, where $\mathfrak{M}^n$ and $\mathfrak{M}_p^n$ mean the n-fold direct products of the spaces $\mathfrak{M}$ and $\mathfrak{M}_p$ respectively. This equation was considered by H. Tanaka [6] and T. Ueno [7], in a slightly different form, to extend the result of McKean [5] and Johnson [4] concerning the propagation of chaos. In [6], the following condition:

$$(3) \qquad \int_{1-\varepsilon}^{1} \frac{d\xi}{\xi - \sum_{n=1}^{\infty} a_n \xi^n} = +\infty \qquad \text{for any} \quad \varepsilon > 0,$$

is assumed to prove the propagation of chaos. This condition seems closely related to the condition of the uniqueness of the solution of (1). In this paper, as a remark to [6], we give an extension of Wild's sum for the solution of the equation (1) and investigate the relation between the condition (3) and the uniqueness of the solution of (1).

1) In this paper, the continuity, differentiability and integral of $u(t)$ are in the sense of topology of weak convergence in $\mathfrak{M}$. In equation (1), we assume the differentiability of $u(t)$ as a matter of course.