198. Oscillatory Property of Certain Non-linear Ordinary Differential Equations. II

By Hirosi ONOSE
Department of Mathematics, Ibaraki University, Mito

(Comm. by Zyoiti SUETUNA, M. J. A., Nov. 12, 1968)

1. The author [2] recently extended certain theorems of Kartsatos [1] and proved that certain even-order differential equations of the form

$$x^{(2n)}+f(t)g(x, x', \cdots, x^{(2n-1)})=0 \tag{1}$$

can have only oscillatory solutions. Non-oscillatory solutions may occur, however, for odd-order differential equations of the same form

$$x^{(2n+1)}+f(t)g(x, x', \cdots, x^{(2n)})=0 \tag{2}$$

when f and g satisfy the same conditions as in the even-order case, and so it becomes a problem how to distinguish oscillatory solutions of the equations (2). (This was pointed out in a discussion with Professors Nakashima and Sugiyama.)

It is indeed possible to prove oscillation theorems even for the equations of the form (2). Roughly speaking, our oscillation theorems assert that any solution of an equation of the form (2) with the same conditions on f and g as in [2] is oscillatory whenever it has at most one zero and n is any positive integer.

Similar oscillation theorems have recently been obtained by Waltman [3] for third order equations under different assumptions.

2. We shall consider the differential equation (2) and the following conditions:

(α) *f is positive function defined on the interval $\bar{I}=[t_0, +\infty)$ with $t_0\geqslant 0$ and $\int_{t_0}^{+\infty} f(t)dt=+\infty$;*

(β) *g is defined on R^{2n+1}; sgn $g(x_1, x_2, \cdots, x_{2n+1})=\operatorname{sgn} x_1$ for any $(x_1, x_2, \cdots, x_{2n+1})\in R^{2n+1}$; and $g(\lambda x_1, \lambda x_2, \cdots, \lambda x_{2n+1})=\lambda^{2p+1}g(x_1, x_2, \cdots, x_{2n+1})$ for any $(x_1, x_2, \cdots, x_{2n+1})\in R^{2n+1}$, and $\lambda\in R$ and some nonnegative integer p;*

(γ) *g is defined on R^{2n+1}; sgn $g(x_1, x_2, \cdots, x_{2n+1})=\operatorname{sgn} x_1$ for any $(x_1, x_2, \cdots, x_{2n+1})\in R^{2n+1}$; $g(-x_1, -x_2, \cdots, -x_{2n+1})=-g(x_1, x_2, \cdots, x_{2n+1})$ for any $(x_1, x_2, \cdots, x_{2n+1})\in R^{2n+1}$; and for any $2\leqslant k\leqslant 2n$ and $c\geqslant 0$, the function $g(x_1, x_2, \cdots, x_{2n+1})$ has a definit limit $G(k, c)$, which is positive or $+\infty$, as $x_1\to+\infty, \cdots, x_{k-1}\to+\infty, x_k\to c, x_{k+1}\to 0, \cdots, x_{2n+1}\to 0$;*

where all functions considered are real-valued and continuous on their domains. Then our theorems read as follows.