# 197. *Inertia Groups of Low Dimensional Complex Projective Spaces and Some Free Differentiable Actions on Spheres. I*

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**1. Introduction and preliminary lemmas.** Sullivan has proved that the concordance classes of smoothing the combinatorial complex projective space is in one-to-one correspondence with the $c$-orientation preserving diffeomorphism classes where $c$ is the generator of $H^2(CP^n)$ (see [6]). The conjugation map $g:(e_0,\cdots,e_n)\to(\bar{e}_0,\cdots,\bar{e}_n)$ (the complex conjugation) induces the diffeomorphism $g:CP^n\to CP^n$ such that $g_*(c)=-c$. Let $s:[CP^n,PD/O]\to\mathcal{S}(CP^n)$ be the natural correspondence from the concordance classes to the smooth structures. If $s(c_1)=CP^n$ (the natural smooth structure) and $s(c_2)=CP'^n$ and if there exists a diffeomorphism $d:CP^n\to CP'^n$ such that $d_*(c)=-c$ (where $c$ is determined by the concordance class), then $(dg)_*(c)=d_*g_*(c)=d_*(-c)=c$, i.e., the composed diffeomorphism $d\cdot g$ induces the $c$-orientation preserving diffeomorphism. This implies that two concordance classes $c_1$, $c_2$ such that $s(c_1)=s(c_2)=CP^n$ are equivalent.

The inertia group of a smooth manifold $M^n$ is interpreted as follows. (For the definition of the inertia group, see [5]). We may assume that the smooth structure $M^n$ corresponds to the zero element $0\in[M,PD/O]$.

**Lemma 1.** $$I(M^n)=(sj)^{-1}(M^n)$$
*where $j$ denotes the homomorphism of the Puppe's exact sequence*

$$\to[M/M\text{-Int}\,D,PD/O]\xrightarrow{j}[M,PD/O]\to[M\text{-Int}\,D,PD/O]\to.$$

Therefore, to study the inertia group $I(CP^n)$, we have only to study the following Puppe's exact sequence,

$$\to[SCP^{n-1},PD/O]\xrightarrow{\partial}[S^{2n},PD/O]\xrightarrow{j}[CP^n,PD/O]\to.$$

Let $f$ be the attaching map $f:\partial e^{2n}\to CP^{n-1}$ of the $2n$-cell $e^{2n}$ in $CP^n$ and $S(f)$ be its suspension map. Then we shall have

**Lemma 2.** $$\partial=\{S(f)\}^*$$
*where $\{S(f)\}^*$ denotes the homomorphism induced by $S(f)$.*

It is well-known that every free differentiable action of $S^1$ (or $S^3$) on a homotopy sphere $\tilde{S}^n$ is always a principal fibration (see [2]) and that this fibration is homotopically equivalent to the classical Hopf fibration (see [4]). Therefore the bundle-theoretic approach to smooth-