195. On Concentric Semigroups*)

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(Comm. by Kenjiro SHODA, M.J.A., Nov. 12, 1968)

By a $\mathfrak{p}$-simple semigroup we mean a semigroup which has no prime ideal, equivalently an $\mathcal{S}$-indecomposable semigroup, i.e., a semigroup which has no semilattice-homomorphic image except a trivial one. (See [5].) A commutative semigroup S is $\mathfrak{p}$-simple if and only if it is commutative archimedean. (See [3], [4].) A commutative archimedean semigroup has one of the following types (cf. [6]):

(1) A commutative nil-semigroup (i.e., some power of every element is zero).

(2) An ideal extension of an abelian group G, $|G|>1$, by a semigroup Z, $|Z|\geqq 1$, of type (1).

(3) A commutative archimedean torsion-free semigroup (i.e., having no idempotent).

Let $C=\bigcap_{n=1}^{\infty} Sa^n$. If S is commutative, C is the kernel (i.e., the minimal ideal) and hence does not depend on a and

$$C=\{0\} \text{ for (1)}; \quad C=G \text{ for (2)}; \quad C=\phi \text{ for (3)}.$$

Let S be a commutative archimedean semigroup and define a relation $\preceq$ by divisibility, i.e.,

$$a \preceq b \text{ iff either } a=b \text{ or } a=bx \text{ for some } x \in S.$$

The relation is a partial ordering on S if and only if either $C=\{0\}$ or $C=\phi$.

Thus C plays an important role in commutative archimedean semigroups. This concept, however, can be defined in any semigroup though it depends on elements. It will be called the "closet" of an element a. In this note we will report some of the results of the study of semigroups with constant closet without proof. The proof will be published elsewhere [7].

Definition. Let S be a semigroup and let $a \in S$.

(4) $C_l(a)=\bigcap_{n=1}^{\infty} Sa^n$ is called the left closet of a in S.

(5) $C_r(a)=\bigcap_{n=1}^{\infty} a^nS$ is called the right closet of a in S.

(6) $C(a)=\bigcap_{n=1}^{\infty} Sa^nS$ is called the closet of a in S.

*) This is a part of the results of the research supported by GP-7608.