234. On the Mixed Problem for the Wave Equation with an Oblique Derivative Boundary Condition

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1. Introduction. On the mixed problem for hyperbolic equations, only few results have been derived. Indeed, up to now, even for second order equations only the problems with the Dirichlet type boundary condition and with the Neumann type boundary condition are studied satisfactorily, and concerning the wave equation we don't know whether the problem with an oblique derivative boundary condition is well posed or not.

In this note, we show that the above problem in a half space is not well posed in L^2-sense.

At first we explain the well-posedness in L^2sense. Let Ω be a sufficiently smooth domain in R^n, L be a second order hyperbolic operator with coefficients in $\mathscr{B}(\Omega\times[0, T])$ and $B=b_1\left(x, t:\frac{\partial}{\partial x}\right)+b_2(x, t)\frac{\partial}{\partial t}$ be a first order differential boundary operator. Consider the mixed problem

$$
(\mathrm{P})\quad\begin{cases}(1.1)\quad L[u(x, t)]=f(x, t) \quad \text{in} \quad \Omega\times(0, T)\\ (1.2)\quad Bu(x, t)=0 \quad \text{on} \quad \partial\Omega\times[0, T]\\ (1.3)\quad u(x, 0)=u_0(x),\ \frac{\partial}{\partial t}(x, 0)=u_1(x).\end{cases}
$$

Definition. *The mixed problem* (P) *is said to be well posed in L^2-sense if for any initial data* $\{u_0(x), u_1(x)\}\in N=\{(u, v): u\in H^2(\Omega),$ $v\in H^1(\Omega)$ *satisfying* $b_1\left(x, 0:\frac{\partial}{\partial x}\right)u+b_2(x, 0)v=0$ *on* $\partial\Omega\}$ *there exists one and only one solution of* (P) *in* $\mathcal{E}_t^0(H^2(\Omega))\cap\mathcal{E}_t^1(H^1(\Omega))\cap\mathcal{E}_t^2(L^2(\Omega))$[1)] *satisfying $L[u]=0$ and the following energy inequality holds for* $t\in[0, T]$.

$$
(1.4)\qquad \|u(x, t)\|_{1,L^2(\Omega)}^2+\left\|\frac{\partial u}{\partial t}(x, t)\right\|_{L^2(\Omega)}^2\leqslant C(\|u_0(x)\|_{1,L^2(\Omega)}^2+\|u_1(x)\|_{L^2(\Omega)}^2).
$$

Then our result is

Theorem. *In the case* $\Omega=\{(x, y): x>0,\ -\infty<y<\infty\}$, $L=\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}$ *and* $B=\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}$, *where b is any non zero real constant, then the mixed problem* (P) *is not well posed in L^2-sense.*

Hereafter we denote by (P_0) the mixed problem for the above Ω,

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1) $f\in\mathcal{E}_t^k(E)$ means that f is k times continuously differentiable in t as E-valued function.