232. On M- and M*-Spaces

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1. In [2], K. Morita has introduced the notion of M-spaces. A topological space X is an M-space if there exists a normal sequence $\{\mathfrak{U}_i \mid i=1,2,\cdots\}$ of open coverings of X satisfying the condition (M) below:

(M) If $\{K_i\}$ is a sequence of non-empty subsets of X such that $K_{i+1}\subset K_i$, $K_i\subset \mathrm{St}(x_0,\mathfrak{U}_i)$ for each i and for some fixed point x_0 of X, then $\bar{K}_i \neq \phi$.

On the other hand, in [1], we introduced the notion of M^*-spaces. A topological space X is an M^*-space if there exists a sequence $\{\mathfrak{F}_i \mid i=1,2,\cdots\}$ of locally finite closed coverings of X satisfying Condition (M), where we may assume without loss of generality that $\mathfrak{F}_{i+1}$ is a refinement of $\mathfrak{F}_i$ for each i. As for the relations between M- and M^*-spaces, the following results are proved by K. Morita [3].

(1) There exists an M^*-space which is locally compact Hausdorff but is not an M-space.

(2) A collectionwise normal space is an M-space if and only if it is an M^*-space.

The first result is a direct consequence of the following (cf. [3]): There is a perfect map $f: X\to Y$ such that X is an M-space but Y is not, and such that X, Y are locally compact Hausdorff spaces. In fact, the space Y is an M^*-space as the image under a perfect map f of an M^*-space X by [1, Theorem 2.3 in I].[1] However, it seems to be unknown whether a normal M^*-space is an M-space or not. The purpose of this paper is to give an affirmative answer for this problem.

2. We shall prove the following main theorem.

Theorem 2.1. *A normal space X is an M-space if and only if it is an M^*-space.*

Before proving Theorem 2.1, we mention a fundamental lemma, which is essentially due to K. Morita [3].

Lemma 2.2. *Let X be an M^*-space with a sequence $\{\mathfrak{F}_i\}$ of locally finite closed coverings of X satisfying Condition* (M), *where $\mathfrak{F}_{i+1}$ is a refinement of $\mathfrak{F}_i$ for each i. Then the following statements are valid.*

(a) *If $\{K_i\}$ is a sequence of non-empty subsets of X such that*

1) In [1, Theorem 2.3 in I], the assumption that X is T_1 is unnecessary.