230. On Proofs of Some Axioms with Sheffer Functor 'D'

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There has been represented an extension of "Fitch's rules" for Sheffer functor by D. F. Siemens [3]. There is a set of four rules for the 'Sheffer stroke' or 'alternative denial', symbolized by '|' or '*D*'. The relation between this functor and other propositional functors, and some deductions from ordinary substitution and detachment rules are shown in K. Iséki [1] and T. W. Scharle [2].

The rule for Introduction (*DI*):

1	p	*H* (hypothesis)
⋮	⋮	
2	q	assumption (i.e., it is assumed that this deduction can be completed)
3	Dqq	assumption
4	Dpp	1, 2, 3, *DI*.

The rule for Elimination (*DE*):

1	Dpq	*H* (given)
2	Dpp	1, *DE* (4)
⋮	⋮	
3	r	assumption
4	Dqq	1, *DE* (2)
⋮	⋮	
5	r	assumption
6	r	2–3, 4–5, *DE*.

The rule for substitution (*DSF*) consists of the following pair:

1	Dpp	given	1	Dpp	given
2	Dpq	1, *DSF*	2	Dqp	1, *DSF*.

The rule for Contraction (*DCF*):

1	$DDppDpp$	given
2	p	1, *DCF*.

The rule for reiteration to the same proof level or to an inner level, '*R*', is also needed. Nicod's rule of transformation can be asserted:

1	p	given
2	$DpDrq$	given
3	q	1, 2, Nicod.

For the details of the proof of this rule, see [3].