229. On Definitions of Boolean Rings and Distributive Lattices

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G. R. Blakley, K. Iséki, and the present author give some new axioms for commutative rings and Boolean rings (see, [1]–[4]).

In this paper, we shall give new characterizations of Boolean rings and distributive lattices.

Theorem 1. *Let* $\langle X, 0, 1, +, ., - \rangle$ *be an algebraic system containing* 0 *and* 1 *as elements of a set* X, *where* $+$ *and* $\cdot$ *are binary operations, and* $-$ *is a unary operation on* X (*we denote* $a\cdot b$ *by* ab). *Then* $\langle X, 0, 1, +, \cdot, - \rangle$ *is a Boolean ring if it satisfies the following conditions*:

1) $r+0=r$,

2) $rl=r$,

3) $((-r)+r)a=0$,

4) $((ar+by)+cz)r=b(yr)+(ar+z(cr))$

for every a, b, c, r, y, z.

It is easily verified that every Boolean ring satisfies 1)–4).

Proof. The proof is divided into the following nine steps.

5) $(-r)+r$

$=((-r)+r)1$ $\{2\}$

$=0.$ $\{3\}$

6) $0a$

$=((-0)+0)a$ $\{5\}$

$=0.$ $\{3\}$

7) $a+b$

$=((a1+b1)+00)1$ $\{2, 1, 6\}$

$=b(11)+(a1+0(01))$ $\{4\}$

$=b+a.$ $\{2, 6, 1\}$

8) cz

$=((01+00)+cz)1$ $\{1, 7, 6, 2\}$

$=0(01)+(01+z(c1))$ $\{4\}$

$=zc.$ $\{6, 1, 7, 2\}$

9) $(b+a)+c$

$=(a+b)+c$ $\{7\}$

$=((a1+b1)+c1)1$ $\{2\}$

$=b(11)+(a1+1(c1))$ $\{4\}$