227. *Pseudo Quasi Metric Spaces*

By Yong-Woon KIM
University of Alberta and Wisconsin State University

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Introduction. Kelly [3] is the first one who studied the theory of bitopological space. A motivation for the study of bitopological spaces is to generalize the pseudo quasi metric space (which we denote as $p-q$ metric). In this paper one observes the relation between $p-q$ metric spaces and the bitopological spaces which are generated by them. In chapter 2, one defines p-complete normal (i.e., pairwise complete normal) space and shows that $p-q$ metric space is p-complete normal. In the last chapter the $p-q$ metrisable problem is considered, and one of the Sion and Zelmer's result [4] is proved directly by a bitopological method. Throughout notations and definitions follow [2] and [3].

Definition. A $p-q$ metric on set X is a non-negative real valued function $p: X\times X\to R$ (reals) such that

(1) $\quad p(x,x)=0$,

(2) $\quad p(x,z)\leq p(x,y)+p(y,z)$ for all $x,y,z\in X$.

In addition, if p satisfies

(3) $\quad p(x,y)=0$ only if $x=y$

then p is said to be a quasi metric. If p satisfies

(4) $\quad p(x,y)=p(y,x)$

with (1) and (2) then p is a pseudo metric. Obviously, if (1), (2), (3), and (4) are satisfied then it is a metric in the usual sense.

Let p be a $p-q$ metric on X and let q be defined by $q(x,y)=p(y,x)$. Then q is a $p-q$ metric on X and q is said to be the conjugate $p-q$ metric of p. We denote the bitopological space X generated by $\{S_p(x,\varepsilon)=\{y\mid p(x,y)<\varepsilon\}\}$ and $\{S_q(x,\varepsilon)=\{y\mid q(x,y)<\varepsilon\}\}$ as (X,P,Q) (see [3]). Throughout this paper (X,L_1,L_2) denotes a bitopological space with topology L_1 and L_2.

(1–2) **Definition** (Kelly [3]). A bitopological space (X,L_1,L_2) is said to be p-normal (i.e., pairwise normal) if for any L_1-closed set A and L_2-closed set B with $A\cap B=\phi$, there exist an L_2-open U and an L_1-open set V such that $A\subset U$, $B\subset V$, and $U\cap V=\phi$.

Kelly [3] defined p-regular bitopological space in an analogous manner.

(1–3) **Definition.** Let (X,L_1,L_2) be a bitopological space,