224. Another Proof of Generalized Sum Theorem for Whitehead Torsion

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Introduction. Kwun and Szczarba [3] established a sum theorem for Whitehead torsion of homotopy equivalence. Their theorem is stated as follows.

Sum Theorem. *Let $f: X \to Y$ be the sum of cellular maps $f_1: X_1 \to Y_1$ and $f_2: X_2 \to Y_2$, where $X = X_1 \cup X_2$, and $Y = Y_1 \cup Y_2$ are finite cell complexes. Suppose that $X_0 = X_1 \cap X_2$ is 1-connected, and that f_1, f_2 and $f_0 = f_1 | X_0 (= f_2 | X_0)$ are homotopy equivalences. Then f is a homotopy equivalence and*

$$\tau(f) = j_{1*}\tau(f_1) + j_{2*}\tau(f_2),$$

where $j_{i*}: \mathrm{Wh}(\pi_1 Y_i) \to \mathrm{Wh}(\pi_1 Y)$ $(i=1, 2)$.

Recently, Hosokawa [1] has generalized this theorem to the case of X_0 being non-simply connected. Indeed, for this general case he obtained the following equality

$$\tau(f) = j_{1*}\tau(f_1) + j_{2*}\tau(f_2) - j_{0*}\tau(f_0),$$

where $j_{0*}: \mathrm{Wh}(\pi_1 Y_0) \to \mathrm{Wh}(\pi_1 Y)$. His proof is accomplished by a geometric idea.

The purpose of this paper is to extend the definition of Whitehead torsion and to prove the generalized sum theorem by making use of the torsions in the extended sense.

For details on the notions of Whitehead group and torsion, we refer the reader to Whitehead [5] and Milnor [4].

§ 1. Let (K, L) be a pair consisting of a finite, connected CW-complex K, and a subcomplex L which is a deformation retract of K. The torsion $\tau(K, L)$ of the pair (K, L) is defined as an element of the Whitehead group $\mathrm{Wh}(\pi_1 K)$, using the chain complex of the universal covering complex of (K, L) [4].

Now let G be a normal subgroup of $\Pi = \pi_1 K$. Then there exists a regular covering $p: (\tilde{K}, \tilde{L}) \to (K, L)$ such that $p_*(\pi_1 \tilde{K}) = G$. $\Pi_G = \pi / G$ operates freely on $(\tilde{K}, \tilde{L})$, and therefore $C_q(\tilde{K}, \tilde{L})$ is a free $Z[\Pi_G]$-module. Since $\tilde{L}$ is a deformation retract of $\tilde{K}$, the chain complex $C(\tilde{K}, \tilde{L})$ is acyclic.

Let $e_1, \cdots, e_\alpha$ denote the q-cells of $K - L$. For each i, choose a representative cell $\tilde{e}_i$ of $\tilde{K}$ lying over e_i. Then $c_q = (\tilde{e}_1, \cdots, \tilde{e}_\alpha)$ can be considered as a basis for the $Z[\Pi_G]$-module $C_q(\tilde{K}, \tilde{L})$. By these prefer-