223. On a Product Theorem in Dimension*)

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1. Let X be a topological space and G an abelian group. The *cohomological dimension* $D(X:G)$ *of* X *with respect to* G is the largest integer n such that $H^n(X, A:G)\neq 0$ for some closed set A of X, where H^* is the Čech cohomology group based on the system of all locally finite open coverings. If X is normal and $\dim X<\infty$, then $D(X:Z)=\dim X$ by [2] and [5, II]. Here $\dim X$ is the covering dimension of X and Z is the additive group of integers.

In this paper we shall show a product theorem for cohomological dimension with respect to certain abelian groups. The theorem is given by proving a product theorem for covering dimension and by applying the same method as developed in [3] and [4]. We use the following groups:

Q=the rational field, Z_p=the cyclic group of order p,

R_p=the subgroup of Q consisting of all rationals whose denominators are coprime with p.

Here p is a prime. Let G be one of the groups Z, Q, R_p, and Z_p, p a prime. We shall show that the relation

$$(*) \qquad D(X\times Y:G)\leqq D(X:G)+D(Y:G)$$

holds if either (i) X is a paracompact Morita space and Y metrizable, or (ii) X is a Lindelöf Morita space and Y a σ-space. See 2 for definition of Morita spaces and σ-spaces. It is well known that the relation $(*)$ is not true for arbitrary groups. Also, the equality $D(X\times Y:G)=D(X:G)+D(Y:G)$ does not generally hold even if G is Q or Z_p, and X and Y are separable metric spaces. Next, let βX be the Stone-Čech compactification of X. If G is finitely generated, then it is known by [5] that $D(\beta X:G)=D(X:G)$. We shall prove that $D(\beta X:G)\geqq D(X:G)$ if X is a paracompact Morita space and G is Q or R_p, p a prime. Throughout the paper all spaces are Hausdorff and maps are continuous.

2. Let $\mathfrak{m}$ be a cardinal number $\geqq 1$. A topological space X is called an $\mathfrak{m}$-*Morita space* if for a set Ω of power $\mathfrak{m}$ and for any family $\{G(\alpha_1, \cdots, \alpha_i)\,|\,\alpha_1, \cdots, \alpha_i\in\Omega;\ i=1, 2, \cdots\}$ of open sets of X such that $G(\alpha_1, \cdots, \alpha_i)\subset G(\alpha_1, \cdots, \alpha_i, \alpha_{i+1})$ for $\alpha_1, \cdots, \alpha_i, \alpha_{i+1}\in\Omega$, $i=1, 2, \cdots,$

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