222. *Remark on Yokoi's Theorem Concerning the Basis of Algebraic Integers and Tame Ramification*

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In this paper we shall prove a theorem (Theorem 1 in the following) which, the author thinks, is essentially a refinement of Yokoi's theorem (Theorem 2 of [2]). From it follows a characterization of tame ramification, which we shall state as Theorem 2.

Theorem 1. *Let k be a finite algebraic number field and K/k be a cyclic extension of prime degree l. Let $\mathfrak{o}$ and $\mathfrak{O}$ be the rings of algebraic integers of k and K. Then we have the following basis x_i, y_i, z_m ($i=1, \cdots, t$, $j=t+1, \cdots, n$, $m=1, \cdots, n(l-1)$) of $\mathfrak{O}$ over the rational integer ring $\boldsymbol{Z}$, i.e.:*

$$\mathfrak{O}=\boldsymbol{Z}[x_1, \cdots, x_t, y_{t+1}, y_n, z_1, \cdots, z_{n(l-1)}]$$

such that $x_1, \cdots, x_t$, $S_{K/k}y_{t+1,\ldots}$, $S_{K/k}y_n$ consist a basis of $\mathfrak{o}$ over $\boldsymbol{Z}$ and $S_{K/k}z_m=0$ for $1\leqq m\leqq n(l-1)$, where $S_{K/k}$ denotes the relative trace of K to k.)*

Let H be the Galois group of K/k. We denote the group ring $\boldsymbol{Z}[H]$ of H over $\boldsymbol{Z}$ by Λ. Obiously $\mathfrak{O}$ is a Λ-module. We consider it as a representation module of H (accordingly of Λ).

Theorem 2. *Let K/k and $\mathfrak{O}$ be as in Theorem 1. Then K/k is tamely ramified at every prime ideal of k if and only if no Λ-module on which H acts trivially appears as a direct summand of $\mathfrak{O}$ (considered as Λ-module).*

At first we state the following well known facts which are useful in the proof of the theorems; let H be a cyclic group of prime order l (for example, the Galois group of K/k stated in the above) and $\Lambda=\boldsymbol{Z}[H]$ be its group ring over $\boldsymbol{Z}$ (as before). Let h be a fixed generator of H and let $\theta=\cos 2\pi/l+i\sin 2\pi/l$, so that θ is a primitive lth root of 1. Let $R=\boldsymbol{Z}[\theta]$. As is shown in [1], there are three and only three classes of indecomposable Λ-modules, i.e.:

i) H-trivial modules, i.e., modules on which H acts trivially.

ii) Taking A to be *a* R-fractional ideal, we may turn A into *a* Λ-module by defining

$$ha=\theta a \text{ for } a\in A.$$

iii) Let y be an indeterminate and A be *a* R-fractional ideal. We

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*) We need not suppose that k and K are absolute Galois number fields, which is different from [2].