9. *Local Knots of 2-Spheres in 4-Manifolds*

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Throughout this paper we will only be concerned from the combinatorial point of veiw. By $(fM^2 \subset M^4)$ we denote a pair of manifolds such that M^4 is a triangulated oriented 4-dimensional manifold and fM^2 is a properly embedded oriented 2-dimensional manifold as a subcomplex in M^4 and f is a piesewise linear embedding of M^2 in M^4.

We measure the *local knot type** of the embedding f at an interior point x of M^2 as follows, [1], [3]. Let $\mathrm{St}(fx, M^4)$ and $\mathrm{St}(x, M^2)$ denote the closed star neighborhoods of fx in M^4 and x in M^2 respectively. The boundary** $S^3 = \partial \mathrm{St}(fx, M^4)$ of $\mathrm{St}(fx, M^4)$ is a 3-sphere with an orientation inherited from that of M^4, and the boundary $S^1 = \partial \mathrm{St}(x, M^2)$ is a 1-sphere with an orientation inherited from that of M^2. The oriented knot type (denote $\kappa(x)$) of the embedding of fS^1 in S^3 is called the *local knot type* of the embedding f at x. When $\kappa(x)$ is of trivial type, we may say that the *local knot type is O* or that fM^2 is *locally flat* (*unknotted*) at fx. A 2-manifold fM^2 is called *locally flat* if it is locally flat at each of its points. When $\kappa(x)$ is of non-trivial type, we may say that fM^2 is *locally knotted* at fx or that fx is locally knotted point of fM^2.

Of course the local knot type can also be measured at a boundary point $x \in \partial M^2$. In this case $\mathrm{cl}(\partial \mathrm{St}(fx; M^4) \cap \mathcal{J} M^4)$ and $\mathrm{cl}(\partial \mathrm{St}(x, M^2) \cap \mathcal{J} M^2)$ are 3-cell and 1-cell respectively and the local knot type is a type of (1, 3)-cell pair. In this paper we shall consider only embeddings whose boundary points are all locally flat (unknotted).

Since a locally knotted point must be a vertex in any triangulation of the pair $(fM^2 \subset M^4)$ the locally knotted points are always isolated. If M^2 is compact, there can be only a finite number of locally knotted points.

R. H. Fox and J. W. Milnor observed "Under which condition can a given collection of knot types $\kappa_1, \cdots, \kappa_n$ be the set of local knot types of some embedding of a 2-sphere S^2 in the 4-space R^4?" and defined the slice knot types and showed that a collection $\kappa_1, \cdots, \kappa_n$ of knot types can occur as the collection of local knot types of a 2-sphere

*) R. H. Fox and J. W. Milnor called it the local *singularity* [1], but as it is confused with the self-intersection (socalled singularity) we'll use this terminology.
**) ∂=boundary, $\mathcal{J}$=interior, cl=closure.