8. A Note on Filipov's Theorem

By Jun-iti NAGATA[1)]
Department of Mathematics, University of Pittsburgh

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V. V. Filipov [1] proved the following theorem:

Filipov's Theorem. *A paracompact M-space with a point-countable open basis is metrizable.*[2)]

On the other hand A. Okuyama [4] proved

Okuyama's Theorem. *A space X is metrizable if and only if it is a paracompact M-space, and the diagonal of the product space $X\times X$ is a G_δ-set.*

These two metrization theorems for an M-space look like to be considerably different, but the fact is that we can easily form a theorem which includes both of them as corollaries.

Theorem. *A space X is metrizable if and only if it is a paracompact M-space with a point-countable collection $\mathcal{U}$ of open sets such that for any different points x and y of X there is $U\in\mathcal{U}$ satisfying $x\in U$ and $y\notin U$.*

Proof. We shall prove only the sufficiency. The proof is a slight modification of Filipov's, and we make a full use of the following Miščenko's theorem [2] as Filipov did:

Miščenko's Theorem. *Let $\mathcal{U}$ be a point-countable collection of subsets of a set X and X' a subset of X. Then there are at most countably many finite minimal covers (=coverings) of X' by elements of $\mathcal{U}$, where we mean by a minimal cover a cover which contains no proper subcover.*

Now let us assume that X is a space satisfying the conditions in the theorem. Since X is a paracompact M-space, there is a metric space Y and a perfect mapping f from X onto Y. Note that for each $x\in X f^{-1}f(x)$ is a compact set of X, and we shall denote this set by F_x throughout this paper. For each natural number n we denote by $\mathcal{V}_n$ a locally finite open cover of Y such that *mesh* $\mathcal{V}_n=$ *sup* {*diameter of*

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2) Actually he used the terminology '*p-space*' instead of '*M-space*', but for a paracompact space the two concepts, M-space (due to K. Morita) and p-space (due to A. Arhangelskii) coincide with each other, and a paracompact M-space is characterized as the inverse image of a metric space by a perfect mapping. As for terminologies and symbols in this paper see J. Nagata [3]. Also note that all spaces in this paper are Hausdorff spaces.