6. *On Zero Entropy and Quasi-discrete Spectrum for Automorphisms*

By Nobuo AOKI
Department of Mathematics, Josai University, Sakado, Saitama

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§ 1. Abramov [1] has defined the notion of an automorphism with quasi-discrete spectrum. Hahn and Parry [7] have developed an analogous theory for homeomorphisms of compact spaces, and Parry [10] has shown that the maximal partition of an ergodic affine transformation of a compact connected metric abelian group and that of the ergodic affine transformation with quasi-discrete spectrum coincide. In §3 we prove that totally ergodic automorphisms belonging to $C_2(T)$ [3] have quasi-discrete spectrum if and only if the automorphisms have zero entropy. The study in this paper depends on [4], [10], and [16].

§ 2. Let (X, Σ, m) be a Lebesgue measure space with normalized measure m. We denote by $\Sigma(m)$ the Boolean σ-algebra by identifying sets in Σ whose symmetric difference has zero measure, and the measure m is induced on the elements of $\Sigma(m)$ in the natural way. Let $L^2(\Sigma)$ be the Hilbert space of complex-valued square integrable functions defined on (X, Σ, m) and let $L^\infty(\Sigma)$ be the Banach space of complex-valued m essentially bounded functions defined on (X, Σ, m) but sometimes we use $L^2(\Sigma(m))$ instead of $L^2(\Sigma)$. Let T be automorphism of (X, Σ, m) and we denote by $V_T : f(x) \rightarrow f(Tx)$ $(x \in X,\ f \in L^2(\Sigma))$ the linear isometry induced by T. T is said to be *totally ergodic* if T^n is ergodic for every positive integer n and to be a *Kolmogorov automorphism* if there exists sub σ-field $\mathscr{B}$ such that (1) $\mathscr{B} \subset T^{-1}\mathscr{B}$ (2) $\bigcap_{n=-\infty}^{\infty} T^n\mathscr{B} = \mathscr{Q}$ ($\mathscr{Q}$ a field whose measurable sets are measure zero or one) and (3) $\bigvee_{n=-\infty}^{\infty} T^n\mathscr{B} = \Sigma$. If there is a basis $\boldsymbol{O}$ of $L^2(\Sigma)$ each term of which is a normalized proper function of T, then T is said to have *discrete spectrum*. Clearly $\boldsymbol{O}$ includs a circle group K. If T is ergodic then it turns out that $|f| = 1$ a.e. for each $f \in \boldsymbol{O}$, and that $\boldsymbol{O} = K \times \boldsymbol{O}(T)$ where $\boldsymbol{O}(T)$ is a subgroup of $\boldsymbol{O}$ isomorphic to the factor group $\boldsymbol{O}/K$. If T is totally ergodic and has discrete spectrum, then $C_1(T) \neq C_2(T) = C_3(T)$ [3]. If T is ergodic and has discrete spectrum, then for every $Q \in C_2(T)$ there exist almost automorphisms W, S such that W has each function of $\boldsymbol{O}(T)$ as a proper function and V_S maps $\boldsymbol{O}(T)$ onto itself, and $Q = WS$ a.e. [3] and [4]. Let T be ergodic, then for an automorphism S satisfying $V_S\boldsymbol{O}(T) = \boldsymbol{O}(T)$