5. On Generalized Commuting Properties of Metric Automorphisms. II

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We study properties of ergodicity, totally ergodicity and mixing for the second class $C_2(T)$ of the generalized T-commuting order when T is ergodic metric automorphism with discrete spectrum. We use notations of [2]. In this paper results were first obtained by Hahn [3]. A metric automorphism S is said to have *continuous spectrum* if the only proper value of V_S is the number one and it is simple, and to have *infinite Lebesgue spectrum* if $L^2(X)$ has an orthonormal base $\{f_{i,n}: n=0,1,2,\cdots; i\in$ [infinite index set]$\}$ where $V_S f_{i,n}=f_{i,n+1}$ a.e. A countable sequence $E_1, E_2, \cdots$ of X is called a *separating sequence* if for every pair of x, y in X with $x\neq y$ there exists an integer n satisfying $x\in E_n$, $y\in X\backslash E_n$. If two automorphisms on a finite measure space (X, Σ, m) which contains a separating sequence $E_1, E_2, \cdots$ of measurable sets induce the same metric automorphism, then they differ on at most a set of measure zero [5]. Let G' be the group of all automorphisms of X with the identity I. We define as in [1], $C'_0(T)=\{S\in G': S=I \text{ a.e.}\}$ and n-th class $C'_n(T)=\{S\in G': T^{-1}S^{-1}TS\in C'_{n-1}(T)\}$, $n=1, 2, \cdots$ of the generalized T-commuting order for an ergodic automorphism T which has discrete spectrum.

Proposition 1. *Let (X, Σ, m) be a finite measure space which contains a separating sequence of measurable sets. If an automorphism T is totally ergodic and has discrete spectrum, then $C'_1(T)\neq C'_2(T)=C'_3(T)$. Furthermore, $C'_0(T)$, $C'_1(T)$, and $C'_1(T)$ are subgroups of G'.*

Proof. We denote by $\tilde{S}:\tilde{E}\to S^{-1}E$($\tilde{E}$ an element of the measure algebra and E a copy of $\tilde{E}$) the metric automorphism on the measure algebra induced by $S\in G'$. Let $\tilde{G}$ be a set $\{\tilde{S}: S\in G'\}$ and let $C_0(\tilde{T})[C_n(\tilde{T}), n=1, 2, \cdots]$ be a set $\{I\}$ a set $[\{\tilde{S}\in\tilde{G}: \tilde{S}\tilde{T}\tilde{S}^{-1}T^{-1}\in C_{n-1}(\tilde{T})\}, n=1, 2, \cdots]$. Then by [2] we see that $C_2(\tilde{T})=C_3(\tilde{T})$, and that $C_0(\tilde{T})$, $C_1(\tilde{T})$, and $C_2(\tilde{T})$ are subgroups of $\tilde{G}$. Since (X, Σ, m) contains a separating sequence of measurable sets, we can conlcude that $C'_2(T)=C'_3(T)$, and that $C'_0(T)$, $C'_1(T)$, and $C'_2(T)$ are subgroup of G'.

Let T be an ergodic metric automorphism with discrete spectrum. Then for every $S_2\in C_2(T)$ there exist metric automorphisms W, S such that W has each function of $\boldsymbol{O}(T)$ as proper function and the linear isometry V_S induced by S maps $\boldsymbol{O}(T)$ onto itself, and $S_2=SW(*)$[2].