3. *Geodesic Flows and Isotropic Flows*

By Nobukazu ÔTSUKI
Department of Mathematics, Kyoto University

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Introduction. The purpose of this paper is to give a unified treatment of geodesic flows, horocycle flows and isotropic flows. It is shown that in the two dimensional case various flows are obtained by perturbation of the geodesic flow and that the entropies of the horocycle flow, the geodesic flow and the constant isotropic flow on an n-dimensional compact Riemannian manifold with constant positive curvature are all equal to zero. Moreover by our method we can prove the theorem of Anosov to the effect that the geodesic flow on the compact Riemannian manifold with negative curvature is a C-flow and furthermore a K-flow [1]. This will be published in another paper.

1. Preparation from geometry. We shall define the maps π_* and K which we need later. For details, consult P. Dombrowski [3]. Let M be an n-dimensional differentiable Riemannian manifold and G be the fundamental tensor of M. TM_p denotes the tangent space at $p \in M$ and $TM = \bigcup_{p\in M} TM_p$ the tangent bundle. If $(x^1, \cdots, x^n)$ is a local coordinate system at $p \in M$ and $v = \sum_{i=1}^n v^i \partial/\partial x^i$, then we can take $(x^1, \cdots, x^n\ v^1, \cdots, v^n)$ as a local coordinate system at $v \in TM$. This shows that TM is a differentiable manifold, and we shall denote TM by W. For projection $\pi : W \to M$, the map $\pi_* : TW \to W$ is naturally defined. A coordinate representation of π_* is of the form $\pi_*(X_v) = \sum_{i=1}^n (\xi^i \partial/\partial x^i)$, where $TW \ni X = \sum_{i=1}^n (\xi^i \partial/\partial x^i + \xi^{i+n}\partial/\partial v^i)$, $W \ni v = \sum_{i=1}^n v^i \partial/\partial x^i$. Next, let us define the map $K : TW \to W$ as follows:

$$K(X_v) = \sum_{i,j,k=1}^n (\xi^{n+i} + \Gamma^i_{jk}\xi^j v^k)\frac{\partial}{\partial x^i},$$

where Γ^i_{jk} are Christoffel's symbol.

By the maps π_* and K, a 2-covariant tensor $\tilde{G}$ on W is defined: $G(X, Y) = G(\pi_* X, \pi_* Y) + G(KX, KY)$, where X, Y are vector fields on W. W is a Riemannian manifold with $\tilde{G}$ as the fundamental tensor.

2. Geodesic flow. We define the geodesic flow with the following vector field S ("geodesic spray") on $W : \pi_* S_v = v$ and $KS_v = 0$, where we should consider $v \in W$ on the left side and $v \in TM$ on the right side.

Put $W_1 = \{v \in W\,;\, \|v\| = 1\}$, then W_1 is a regular submanifold of W. We can show that W_1 is an invariant manifold of S. The geodesic flow restricted to W_1 will be called simply the *geodesic flow* on M in the following. We note that it preserves the Riemannian measure on