# 2. A Note on the Metrizability of M-Spaces

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The notion of an $M$-space was introduced by Morita in [6] and in [1] Okuyama gave conditions for an $M$-space to be metrizable. Recently Borges, in [2], generalized some of Okuyama's work by considering $w\Delta$-spaces. In this note a condition is given under which a $w\Delta$-space is a Moore space.

The terminology of [4] will be used except all spaces will be $T_1$.

Definition 1. A space $X$ is said to be a $w\Delta$-space if there exists a sequence $\{B_1, B_2, \cdots\}$ of open covers of $X$ such that for each $x_0 \in X$, if $x_n \in \mathrm{St}(x_0, B_n)$ for each natural number $n$, then the sequence $\{x_1, x_2, \cdots\}$ has a cluster point.

Definition 2. A space $X$ is said to be an $M$-space provided there exists a normal sequence[1] of open coverings of $X$ satisfying the following: If $\{A_1, A_2, \cdots\}$ is a sequence of subsets of $X$ with the finite intersection property and if there exists $x_0 \in X$ such that for each natural number $n$ there exists some $A_k \subset \mathrm{St}(x_0, B_n)$, then

$$\bigcap_{i=1}^{\infty} A_i^- \neq \emptyset.$$

Clearly all metrizable or countably compact spaces are $M$-spaces. In [2], Borges shows that each $M$-space is also an $w\Delta$-space.

Definition 3. Let $X$ be a regular space. Then $X$ is a quasi-developable space if there exists a sequence $\{G_1, G_2, \cdots\}$ of collections of open subsets of $X$ such that if $x \in X$ and $R$ is an open subset of $X$ containing $x$, then there is a natural number $n(x, R)$ such that some element of $G_{n(x,R)}$ contains $x$ and each member of $G_{n(x,R)}$ that contains $x$ lies in $R$. The sequence $\{G_1, G_2, \cdots\}$ is called the quasi-development for $X$.

Notice that if, in Definition 3, it is also required that each $G_i$ be a cover for $X$, then $X$ satisfies the first three parts of Axiom 1 of [5] and $X$ is called a Moore space. In this case $\{G_1, G_2, \cdots\}$ is called a development for $X$.

Quasi-developable spaces are investigated extensively in [1] where

1) A sequence $\{U_1, U_2, \cdots\}$ of open covers of a topological space $X$ is a normal sequence if for each natural number $n$ $\mathrm{St}(x, U_{n+1})$ is contained in some element of $U_n$, for each $x \in X$.