1. *Maximal Sum-Free Sets of Elements of Finite Groups*

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1. *Introduction.* Let G be an additive group. If S and T are non-empty subsets of G, we write $S \pm T$ for $\{s \pm t\,;\, s \in S,\ t \in T\}$ respectively, $|S|$ for the cardinal of S and $\bar{S}$ for the complement of S in G. We abbreviate $\{f\}$, where $f \in G$ to f. We say that S is sum-free in G if S and $S+S$ have no common element and that S is maximal sum-free in G if S is sum-free in G and $|S| \geqq |T|$ for every T sum-free in G. We denote by $\lambda(G)$ the cardinal of a maximal sum-free set in G. We say that S is in arithmetic progression with the difference d if $S=\{s, s+d, s+2d, \cdots, s+nd\}$ for some s and $d \in G$ and some integer $n \geqq 0$.

In [3] Yap obtained certain results concerning $\lambda(G)$ for abelian G. The main purpose of this paper is to generalize and to improve, where possible, his results.

2. *Abelian groups.* Throughout this section G is an abelian group. We use the following theorem [2; p. 6] due to M. Kneser:

Theorem 1. *Let A and B be finite non-empty subsets of G. Then a subgroup H of G exists such that $A+B+H=A+B$ and $|A+B| \geqq |A+H| + |B+H| - |H|$.*

Suppose that S is a maximal sum-free set in G. Then a subgroup H of G exists such that

$$S+S+H=S+S \quad \text{and} \quad |S+S| \geqq 2|S+H| - |H|. \tag{1}$$

Lemma 1. *$S+H$ is also a sum-free set in G.*

Proof. Otherwise, $S+H$ and $(S+H)+(S+H)=S+S$ have a common element. Thus $s+h=s_1+s_2$ for some s, s_1 and $s_2 \in S$ and some $h \in H$. Hence $s=s_1+s_2-h \in S+S+H=S+S$. This is not possible since S is sum-free in G.

It now follows that $S+H=S$ since S is maximal sum-free in G. Thus we have

Lemma 2. *S is a union of cosets of H in G.*

Hence $|H|$ is a divisor of $|S|$. Now $|G| \geqq |S| + |S+S| \geqq 3|S| - |H|$, from (1). Hence

$$|S| \leqq |H| \left[\frac{1}{3} \left(\frac{|G|}{|H|} + 1 \right) \right],$$

where $[x]$ denotes the integer part of x. Thus