25. Note on Embeddings of Lens Spaces

By Ryosuke NAKAGAWA
Tokyo University of Education

(Comm. by Kinjirô KUNUGI, M. J. A., Feb. 12, 1969)

An embedding or immersion of M^n in R^{2n-k} is said to have *efficiency* k (here M^n is an n-dimensional manifold). In [1] and [2] Mahowald and Milgram gave excellent results on efficiency of projective spaces. Their methods are applicable for lens spaces and by using the results in [3] and [4], we can obtain some results on efficiency of embeddings of lens spaces.

Let p be any odd integer ≥ 3 and $q_0, \cdots, q_n$ be integers relatively prime to p. The cyclic group Γ of order p with generator t acts on the sphere $S^{2n+1} \subset C^{n+1}$ as follows;

$$t^k(z_0, \cdots, z_n) = (\theta^{kq_0} z_0, \cdots, \theta^{kq_n} z_n),$$

where $(z_0, \cdots, z_n)$ is a complex $(n+1)$-tuple representing a point of S^{2n+1} and $\theta = \exp(2\pi i/p)$. The orbit manifold S^{2n+1}/Γ is a lens space $L^n = L^n(p; q_0, \cdots, q_n)$.

Let $L^m = L^m(p; q_0, \cdots, q_m) \subset L^{n+m+1} = L^{n+m+1}(p; q_0, \cdots, q_{n+m+1})$ be the subspace with the last $n+1$ coordinates 0, while $L^n = L^n(p; q_{m+1}, \cdots, q_{n+m+1})$ is the subspace having the first $m+1$ coodinates 0. A vector bundle $L^{n+m+1} - L^n$ over L^m will be denoted by $L_{n+m+1,m}$.

Let a be an integer such that $a = 4b + c$, $0 \leqq c \leqq 3$, then $j(a) = 8b + 2^c$, and if $d+1 = 2^a e$ with e odd we set $K(d) = j(a) - 1$.

Proposition 1. *Suppose there are differentiable embeddings* $f: L^n \subset R^\alpha$, $g: L^m \subset R^\beta$, $h: L_{n+m+1,m} \subset R^{\beta+\sigma}$ *so that either* (i) $\beta+\sigma > 2(2m+1)$ *or* (ii) $\beta+\sigma = 2(2m+1)$ *and* $2(n+1) \leqq K(2m+1)$, *then if the normal bundle* η_f *of the embedding* f *has* σ *trivial sections, there is a topological embedding* $L^{n+m+1} \subset R^{\alpha+\beta+1}$.

This can be proved by the same way in [1].

Proposition 2. *There are embeddings* $f: L^1 \subset R^6$, $g: L^3 \subset R^{14}$, $h: L^n \subset R^{2(2n+1)} (n \neq 1, 3)$ *and* η_f, η_g, η_h *have* 2, 4, $K(2n+1)$ *sections respectively.*

This is proved by Theorem 2.2 in [1] and (4.1) in [4].

Proposition 3.

(1) *If* $2n \geqq m$, $L_{n+m+1,m} \subset R^{3m+2n+3+\varepsilon}$, $\varepsilon = \frac{1}{2}(1+(-1)^m)$.

(2) *If* $2n < m$, $L_{n+m+1,m} \subset R^{4m+3}$.

(3) *If* $2n < m$ *and* $2(n+1) \leqq K(2m+1)$, $L_{n+m+1,m} \subset R^{4m+2}$.

It is shown in [3] that $L_{n+m+1,m} \subseteq R^{3m+2n+3+\varepsilon}$ and hence we have (1)