23. Lesniewski's Protothetics S1, S2. I

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The systems S1 and S2 are defined originaly by S. Lesniewski [1]. The definitions, theorems and some relations between S1 and S2 are also shown by K. Iséki [2]. The equivalences of some laws in S1 are proved by K. Chikawa [3].

In this paper we shall prove that every theorem of S2 is a theorem of S1.

Definition. The system which has equivalence as its only primitive term, the following propositions S2A1-S2A4 as its axioms and in which are valid the rule of inference specified below, shall be called the system S2:

(a) the rule of substitution,

(b) the rule of detachment: if α and $\alpha \equiv \beta$ are both theorems of S2, then β is a theorem of S2;

(c) the rule for the distribution of a general quantifier preceeding an equivalence: if $[f, \cdots, g]\{\alpha \equiv \beta\}$ is a theorem of S2, then $[f, \cdots, g]\{\alpha\} \equiv [f, \cdots, g]\{\beta\}$ is a theorem of S2;

(d) the rule of extentionality: any equivalential law of extentionality, i.e.,

$$[f, g]\{(f \equiv g) \equiv [\varphi]\{\varphi(f) \equiv \varphi(g)\}\}$$

is a theorem of S2;

(e) the rule of definition: any correctly built definition is a theorem of S2. Of course, the definitions of S2 consist of equivalence.

S2A1 $[p, q, r]\{(p \equiv q) \equiv ((r \equiv q) \equiv (p \equiv r))\}$,

S2A2 $[p, q]\{(p \equiv q) \equiv [f]\{f(p) \equiv f(q)\}\}$,

S2A3 $[p,q]\{(p \equiv q) \equiv [f]\{(f(p) \equiv f(q)) \equiv (p \equiv q)\}\}$,

S2A4 $[f]\{f([p]\{p\}) \equiv (f([p]\{p\} \equiv [p]\{p\}) \equiv [q]\{f([p]\{p\}) \equiv f(q)\})\}$.

Definition. The system which has implication as its only primitive term, the following proposition A1 as its axiom, and in which are valid the rule of inference specified below, shall be called the system S1;

A1 $[f, g]\{f([p]\{p \supset p\}) \supset (f([p]\{p\}) \supset f(q))\}$.

(a) the rule of substitution:

(b) the rule of detachment: if α and $\alpha \supset \beta$ are both theorems of that system S1, then β is a theorem of S1;

(c) the rules for the general quantifier: the first allows to add