32. *Mappings and M-Spaces*

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Let us recall some of the interesting theorems on metric spaces and compact spaces in relation with maps (=mappings). (As for the references and proofs of these theorems as well as terminologies and symbols, see J. Nagata [4] and [5]. All spaces are at least Hausdorff, and all maps are continuous in the present paper unless the contrary is explicitly mentioned.)

1. A T_1-space, not necessarily Hausdorff, is the image of a metric space by an open continuous map iff (=if and only if) it is 1-st countable (V. Ponomarev-S. Hanai).

2. Every metric space with weight $|A|$ (=the cardinality of the set A) is the image of a subset of Baire's 0-dimensional space $N(A)$ (=the product of countably many copies of the discrete space A) by a perfect map. (K. Morita)

3. Every compact (Hausdorff) space with weight $|A|$ is the continuous image of a closed set of the cantor discontinuum $D(A)$. (P. S. Alexandroff)

4. Every metric space with weight $|A|$ is homeomorphic to a subset of generalized Hilbert space $H(A)$. (C. H. Dowker)

5. Every compact space with weight $|A|$ is homeomorphic to a closed subset of the product of the copies I_α, $\alpha \in A$, of the unit interval $[0, 1]$. (*A*. Tychonoff-P. Urysohn)

As well known, the concept of M-space (paracompact M-space) is an important generalization of that of metric space as well as countably compact space (compact space). Therefore it is natural to try to extend the above theorems to M-spaces and paracompact M-spaces. The purpose of the present paper is to continue our study along this line which started in our previous paper [5].

Theorem 1. *A regular space Y is a q-space in the sense of E. Michael* [1] *iff there are an M-space X and a continuous open* map f *from X onto Y.*

Proof. Sufficiency directly follows from the condition satisfied by X and Y by use of Lemma 1 of [5]. To prove necessity we should note that a regular space is a q-space iff each point has a sequence $U_1, U_2, \cdots$ of open nbds (=neighborhoods) such that $U_1 \supset \bar{U}_2 \supset U_2 \supset \bar{U}_3 \supset \cdots$ and such that if $x_i \in U_i$, $i=1, 2, \cdots$, then $\{x_i \mid i=1, 2, \cdots\}$