30. A Note on Radicals of Ideals in Nonassociative Rings

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Let R be a nonassociative ring and let $\mathfrak{A}=\{u=\mathfrak{P}_n^{(\nu)}\}$ be the set of all formal nonassociative products.[1] In [3], Brown-McCoy has defined that an ideal[2] P is a u-prime ideal, if whenever $u(A_1, \cdots, A_n)$ is contained in P for ideals A_i of R, then at least one of the ideals A_i is contained in P. We shall generalize this concept as follows: Let $\mathfrak{U}$ be any fixed subset of $\mathfrak{A}$. *An ideal P is said to be $\mathfrak{U}$-ideal if whenever $\Sigma_{\mathfrak{P}_n^{(\nu)}\in\mathfrak{U}}\mathfrak{P}_n^{(\nu)}(A_{\nu 1}, \cdots, A_{\nu n})$ is contained in P, where Σ denotes the restricted sum and $A_{\nu i}$ are ideals, then $A_{\nu i}$ is contained in P for some ν, i.* It is the aim of this paper to investigate $\mathfrak{U}$-ideals and to present some related results.

In section 1, $\mathfrak{U}$-systems are defined by analogy with m-systems introduced in [4]. If A is an ideal of R, a $\mathfrak{U}$-radical $\mathfrak{U}(A)$ of the ideal A is defined to be the set of all elements r of R with the property that every $\mathfrak{U}$-system which contains an element of A. We shall prove that $\mathfrak{U}(A)$ is the intersection of all $\mathfrak{U}$-ideals which contains A. Section 2 lays definitions of $\mathfrak{U}^*$-ideals and $\mathfrak{U}^*$-radicals of ideals which are analogous to those of u^*-prime ideals and u^*-radicals of ideals in [3]. We shall show that always $\mathfrak{U}(A)=\mathfrak{U}^*(A)$ under the assumption that $\mathfrak{U}$ is a finite subset of $\mathfrak{A}$, where $\mathfrak{U}^*(A)$ is the $\mathfrak{U}^*$-radical of an ideal A. In the fininal section we define a $\mathfrak{U}$-radical of the ring R, which is denoted by $\mathfrak{U}(R)$, as the one of the zero ideal of R, and show that $\mathfrak{U}(R)$ has the usual properties expected of a radical. Moreover we shall show that $\mathfrak{U}(R_n)=(\mathfrak{U}(R))_n$, where R_n and $(\mathfrak{U}(R))_n$ are the total matric rings of order n with coefficients in R and $\mathfrak{U}(R)$ respectively.

1. $\mathfrak{U}$-ideals and $\mathfrak{U}$-radicals.

Throughout this paper, we let $\mathfrak{U}$ be any fixed subset of $\mathfrak{A}$. The principal ideal generated by an element a of R will be denoted by (a). The complement of an ideal in R will be denoted by $C(A)$.

Lemma 1. *Let P be an ideal of R. Then the following three conditions are equivalent:*

(i) *P is a $\mathfrak{U}$-ideal.*

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1) Following Behrens [1, 2], we shall denote by $\mathfrak{P}_n^{(\nu)}(A_1, \cdots, A_n)$ a fixed type ν of the product of ideals $A_1, \cdots, A_n$ in R.

2) The word "ideal" will always mean a "two-sided ideal."