60. *Structure Theorems for Some Classes of Operators*

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1. We consider bounded linear operators on a Hilbert space H. Denote by $\sigma(T)$, $\sigma_p(T)$, $\sigma_r(T)$, $\sigma_c(T)$ the spectrum, the point spectrum, the residual spectrum and the continuous spectrum respectively, by $r(T)=\sup\{|\lambda| : \lambda\in\sigma(T)\}$ the spectral radius and by $W(T)=\{(Tx, x) : \|x\|=1\}$ the numerical range. It is known [3] that $W(T)$ is convex and conv $\sigma(T)\subset \mathrm{cl}\, W(T)$ (conv = convex hull, cl = closure). An operator T is said to be hyponormal if $T^*T-TT^*\geqslant 0$, or equivalently if $\|T^*x\|\leq\|Tx\|$ for every $x\in H$. As in [1] an operator is said to be restriction-convexoid (reduction-convexoid) if the restriction of T to every invariant (invariant under T and T^*) subspace is convexoid, where convexoid means that conv $\sigma(T)=\mathrm{cl}\, W(T)$.

In this Note we give some theorems on structure of hyponormal and restriction-convexoid operators whose spectrum lies on a convex curve.

2. Our main result in this section is

Theorem 1. *If T is a hyponormal operator and has the following properties*

1° $T^p=ST^{*p}S^{-1}+C$ *for some S for which $o\,\bar{\in}\,\mathrm{cl}\, W(S)$ and C = compact operator*

2° *if* $\mu, \lambda\in\sigma(T)$, $1+\dfrac{\lambda}{\bar{\mu}}+\left(\dfrac{\lambda}{\bar{\mu}}\right)^2+\cdots+\left(\dfrac{\lambda}{\bar{\mu}}\right)^{p-1}\neq o$

then T is a normal operator.

For the proof we need the following

Lemma 1. *If T is a hyponormal operator which is the sum of a self-adjoint operator A and a compact operator C, then T is a normal operator.*

Proof. Since T is hyponormal it is known [10] that T can be expressed uniquely as a direct sum $T=T_1\oplus T_2$ defined on a product space $H=H_1\oplus H_2$ where H_1 is spanned by all the proper vectors of T such that: (a) T_1 is normal and $\sigma(T_1)=\mathrm{cl}\,\sigma_p(T)$, (b) T_2 is hyponormal and $\sigma_p(T_2)=\varnothing$, (c) T is normal if and only if T_2 is normal.

From the fact that $T=A+C$ we conclude by Lemma 2 [10] that $\sigma_c(T_2)\subset\sigma(A)$ and therefore $\sigma_c(T_2)$ is real. Since $\sigma_r(T)$ is open [9] and (T) is closed, we have that $\partial_r(T_2)\subset\sigma_p(T_2)\cup\sigma_c(T_2)=\sigma_c(T_2)$ (∂ = boundary). Therefore T_2 is selfadjoint since T_2 is hyponormal with real spectrum.