59. A Geometric Condition for Smoothability of Bounded Combinatorial Manifold

By Kazuaki KOBAYASHI
Kobe University

(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1969)

1. Introduction. If we modify the paper [3] as follows, then Smoothability Theorem of that paper can be extended the case of bounded combinatorial manifold. For general terminology and definition, see [3].

Let M be a compact bounded combinatorial n-manifold piecewise linearly imbedded in a combinatorial $(n+k)$-manifold W^{n+k} without boundary and X, Y, Z be simplicial divisions of $M, \partial M, W$ such that X and Y are subcomplexes of Z and X respectively. Then $N(X, Z) \bmod Y$ denotes the star neighborhood of X in $Z \bmod Y$, that is, the polyhedron consists of simplices of Z containing simplices whose interior is contained in $|X-Y|$.

Definition 1. Let M be a compact bounded n-manifold imbedded piecewise linearly in euclidean $(n+k)$-space R, $k \geqq 1$. We say that M is *in smoothable position* in R if the following is satisfied.

Let K_0 and L_0 be simplicial divisions of M and R respectively, where K_0 is a complete subcomplex of L_0. And let H_0 be simplicial division of ∂M, where H_0 is a complete subcomplex of K_0.

Then there exist piecewise linear proper imbeddings

$$\varphi_i : M_i \to \partial(N(K_i', L_i') \bmod H_i') - \operatorname{Int} N(H_i', \partial(N(K_i', L_i') \bmod H_i')),$$

for each $0 \leqq i \leqq k-1$, where $M_0 = M$ and for $1 \leqq i \leqq k$, $M_i = \varphi_{i-1}(M_{i-1})$ and where K_i, H_i, and L_i are simplicial subdivisions of $M_i, \partial M_i$ and $\partial(N(K_{i-1}', L_{i-1}') \bmod H_{i-1}') - \operatorname{Int} N(H_{i-1}', \partial(N(K_{i-1}', L_{i-1}') \bmod H_{i-1}'))$.

In the text, however, W_i stands for

$$\partial(N(K_{i-1}', L_{i-1}') \bmod H_{i-1}') - \operatorname{Int} N(H_{i-1}', \partial(N(K_{i-1}', L_{i-1}') \bmod H_{i-1}'))$$

and L_i will be the subcomplex of L_{i-1}' covering W_i for each $1 \leqq i \leqq k$.

Then $\partial W_i = \partial N(H_{i-1}', \partial(N(K_{i-1}', L_{i-1}') \bmod H_{i-1}'))$.

Note that M_i is a combinatorial n-manifold with boundary, which is combinatorially equivalent to M, and W_i is a combinatorial $(n+k-i)$-manifold with boundary, for each $1 \leqq i \leqq k$, satisfying $M_i \subset W_i$ and $W_1 \supset W_2 \supset \cdots \supset W_k$. Furthermore $N(K_0', L_0') \bmod H_0'$ is a regular neighborhood of $M \bmod \partial M$ in R^{n+k} and $N(K_i', L_i') \bmod H_i'$ is a regular neighborhood of $M_i \bmod \partial M_i$ in W_i in the sense of [1], $i=1$.

The extended result of [3] is the following.