58. *Leśniewski's Protothetics S1, S2. III*

By Shôtarô TANAKA

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In this paper, we shall prove that every theorem of S2 is a theorem of S1.

Lemma 8. *The proposition*

(16) $[f]\{f(0)\equiv(f(1)\equiv[q]\{f(0)\equiv f(q)\})\}$

is a theorem of S1.

Proof. We add to S1 the following definitions:

D12 $[f, p]\{\psi_1\langle f\rangle(p)\equiv(f(0)\supset f(p))\}$,

D13 $[f, p]\{\psi_2\langle f\rangle(p)\equiv(f(p)\supset f(0))\}$.

T84 $[f]\{f(0)\cdot f(1)\supset[q]\{f(0)\supset f(q)\}\}$

Proof.
(1) $f(0)$
(2) $f(1)\supset$
(3) $f(0)\supset f(1)$ (T2 ; 1)
(4) $f(0)\supset f(0)$ (T2 ; 2)
(5) $\psi_1\langle f\rangle(1)$ (D12 ; 3)
(6) $\psi_1\langle f\rangle(0)$ (D12 ; 4)
(7) $[q]\{\psi_1\langle f\rangle(q)\}$ (A1′ ; 6 ; 5)
(8) $[q]\{f(0)\supset f(q)\}$ (D12 ; 7)

T85 $[f]\{f(0)\cdot f(1)\supset[q]\{f(q)\supset f(0)\}$

Proof.
(1) $f(0)$
(2) $f(1)\supset$
(3) $f(1)\supset f(0)$ (T2 ; 1)
(4) $f(0)\supset f(0)$ (T2 ; 2)
(5) $\psi_2\langle f\rangle(1)$ (D13 ; 3)
(6) $\psi_2\langle f\rangle(0)$ (D13 ; 4)
(7) $[q]\{\psi_2\langle f\rangle(q)\}$ (A1′ ; 5 ; 6)
(8) $[q]\{f(q)\supset f(0)\}$ (D13 ; 7)

T86 $[f]\{f(0)\cdot f(1)\supset[q]\{f(0)\equiv f(q)\}$ (T84 ; T85 ; T10 ; def (2), ii)

T87 $[f]\{f(0)\supset(f(1)\supset[q]\{f(0)\equiv f(q)\})\}$ (T86)

T88 $[f]\{f(0)\cdot[q]\{f(0)\equiv f(q)\}\supset f(1)\}$

Proof.
(1) $f(0)$
(2) $[q]\{f(0)\equiv f(q)\}\supset$
(3) $f(0)\equiv f(1)$ (2 ; rule(a))
(4) $f(0)\supset f(1)$ (def (2) ; T8 ; 3)
(5) $f(1)$ (4 ; 1)

T89 $[f]\{f(0)\supset([q]\{f(0)\equiv f(q)\}\supset f(1))\}$ (T88)

T90 $[f]\{f(0)\supset(f(1)\equiv[q]\{f(0)\equiv f(q)\})\}$ (def (2), ii ; T87 ; T89)