55. On Limit Spaces and the Double Weak Limit. I

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§1. Introduction. 1.0. Our purpose is to construct the limit spaces (i.e. generalized topological spaces [2] p. 273) J_w, J_{pw} and J_{sep} defined on the set $\tilde{J}$ shown in **1.2** which characterize the generalized double weak limits (itself or with the restriction on sign) expressed by filter. These spaces J_w, J_{pw}, J_{sep} and another space $J_\wedge$ also show the difference among the conditions which characterize the (topological) limit space.

1.1. Let E be a set. Let τx (by τ) be the set of filters defined on the set E corresponding to $x \in E$. We show here the following properties of τx $(L^1) \sim (L^4)$ [2] p. 273, [3] pp. 451–452.

(L^1) τx for any $x \in E$ is a $\wedge$ ideal. Here $\wedge$ ideal is the set of filters satisfying the following conditions (i) (ii);

(i) $\mathfrak{F}_1 \cap \mathfrak{F}_2 \equiv \{F \cup G\,;\, F \in (\mathfrak{F}_1),\, G \in (\mathfrak{F}_2)\} \in \tau x$ for any $\mathfrak{F}_1, \mathfrak{F}_2 \in \tau x$,

(ii) all filters $\mathfrak{F}$ finer than $\mathfrak{F}_1 \in \tau x$ (i.e. $(\mathfrak{F}) \supseteq (\mathfrak{F}_1)$ holds) are also the elements of τx. Here $(\mathfrak{F}_1)$, $(\mathfrak{F}_2)$ and $(\mathfrak{F})$ are the sets consisting of the elements of $\mathfrak{F}_1$, $\mathfrak{F}_2$, and $\mathfrak{F}$ respectively.

Hereafter let $[x]$ denote the filter with the base $\{x\}$, and let $[\mathfrak{V}(x)]$ denote the weakest filter in τx (if it exists).

(L^2) τx for any $x \in E$ contains $[x]$.

(L^3) τx for any $x \in E$ contains $[\mathfrak{V}(x)]$.

(L^4) Corresponding to a $V \in [\mathfrak{V}(x)]$ there exists an element $W (\subseteq V)$ of $[\mathfrak{V}(x)]$ such that $V \in [\mathfrak{V}(y)]$ holds for all $y \in W$.

If τ satisfies (L^1) (L^2), (E, τ) is called a limit space [2] p. 273. If τ satisfies $(L^1) \sim (L^3)$, (E, τ) is called a principal ideal limit space. If τ satisfies $(L^1) \sim (L^4)$, (E, τ) is called a topological space. Limit space is L space by M. Frechet described by the filter. The following (T_1) (T_2) are the axioms of separation in limit space. $(T_1)[x] \bar{\in} \tau y$ holds for any two distinct elements x, y in E. (T_2) $\tau x \cap \tau y = \phi$ holds for any two distinct elements x, y in E.

Let (E, τ) be a limit space. If $\mathfrak{F} \in \tau x$, we call that $\mathfrak{F}$ tends to $x \in E$ by τ, and that x is the limit from $\mathfrak{F}$ by τ. If $[\{x_i\,;\, i \geqslant n\}\,;\, x_i \in E]$ becomes the base of a filter $\mathfrak{F} \in \tau x$, we say that $\{x_n\}$ tends to x by τ. Let A be a set in E. $\bar{A}$ (the closure of A) consists of the points $x \in E$ such that there exists a filter $\mathfrak{F} \in \tau x$ satisfying $F \cap A \neq 0$ for any $F \in (\mathfrak{F})$.