54. A Remark on the Theorem of Bishop

By Chikara WATANABE
Department of Mathematics, Faculty of Science, Kanazawa University

(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1969)

1. On normality of a family of pure-dimensional analytic sets in a domain of C^n, the following theorem of Oka [4] is well-known.

Theorem of Oka. *Let F be a family of pure-dimensional analytic sets in a domain of C^n. Then F is analytically normal if and only if the volumes of elements of F are locally uniformly bounded.*

This theorem was proved by T. Nishino [3] in the case of two variables. The proof of this theorem in the case of n variables was given in our former paper (Watanabe [6]).

On the other hand, the concept of geometric convergence was introduced by E. Bishop as follows.

Let $\{S_\nu\}$ be a sequence of closed subsets in a domain of C^n. It is said that $\{S_\nu\}$ converges geometrically to a closed set S if for any compact set K, $\{S_\nu \cap K\}$ is a convergent sequence in $\mathrm{Comp}(K)$[1] and $S=\bigcup_K \lim(S_\nu \cap K)$ where K ranges over the compact sets. Further Bishop [1] proved the following.

Theorem of Bishop. *Let $\{S_\nu\}$ be a sequence of purely λ-dimensional analytic sets in a domain D of C^n. Suppose that $\{S_\nu\}$ converges geometrically to a closed set S in D. If the volumes of S_ν are uniformly bounded, then S is also an analytic set in D.*

We shall prove that in the above theorem of Bishop, S is also purely λ-dimensional if S is not empty.

2. Let $D=\Delta\times\{|w|<R\}$ be a domain of C^{n+1}, where Δ is a domain of $(z_1, \cdots, z_n)$-space $C^n(z)$. Then the following proposition is well-known (for example, Fujita [2]).

Proposition. *Let S be a purely λ-dimensional analytic set in D. Assume that S is contained in $\Delta\times\{|w|<R_0\}$ for some positive number $R_0<R$. Then the projection of S on Δ is also purely λ-dimensional analytic set in Δ.*

It follows from this:

Corollary. *Let $D=\Delta\times\{|w_1|<R\}\times\cdots\times\{|w_\mu|<R\}$ be a domain of $C^{\lambda+\mu}$ and S be a purely λ-dimensional analytic set in D. If S is contained in $\Delta\times\{|w_1|<R_0\}\times\cdots\times\{|w_\mu|<R_0\}$ for some positive number $R_0<R$, then $\mathfrak{A}=(S, \pi, \Delta)$ is an analytic cover, where π is a projection.*

---

1) For a definition of Comp (K), see [5].