85. On Certain Mixed Problem for Hyperbolic Equations of Higher Order. II

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1. Introduction and results. In the present note, we will extend our results stated before ([3]).

Let Ω be a domain with a bounded boundary Γ of $\boldsymbol{R}^n$. Here we consider a strongly hyperbolic equation

$$(1)\qquad Lu=\left(\frac{\partial^{2m}}{\partial t^{2m}}+a_1(x, D)\frac{\partial^{2m-1}}{\partial t^{2m-1}}+\cdots+a_{2m}(x, D)\right)u$$
$$+(\text{lower order terms})u=f,$$
$$a_k(x, D)=\sum_{|\alpha|=k} a_\alpha(x)D^\alpha,\quad D_j=\frac{1}{\sqrt{-1}}\frac{\partial}{\partial xj},$$

and let all of the roots $\tau_i(x, \xi)$ $(i=1, 2, \cdots, 2m)$ with respect to τ of the equation $\tau^{2m}+a_1(x, \xi)\tau^{2m-1}+\cdots+a_{2m}(x, \xi)=0$ be pure imaginary and distinct mutually, not zero uniformly for $x \in \bar{\Omega}$, $|\xi|=1$.

Here we assume that, after applying any coordinate transformation $(U\cap\Omega, \Gamma\cap\Omega)\mapsto(\boldsymbol{R}^n_+=\{y\in\boldsymbol{R}^n\,|\,y_n>0\}, \{y\,|\,y_n=0\})$ such that on the boundary the conormal direction of a given uniformly strongly elliptic operator $a(x, D)$ of order 2 is changed into the normal direction, the coefficients of the principal part of (1) containing odd power of $\dfrac{\partial}{\partial y_n}$ are zero on the boundary $y_n=0$.

Then we obtain the following

Theorem. *For any* $f(t, x)\in C^1([0, T], L^2(\Omega))$ *and for any initial conditions* $\left(u(0, x), \dfrac{\partial u}{\partial t}(0, x), \cdots, \dfrac{\partial^{2m-1}u}{\partial t^{2m-1}}(0, x)\right)\in D(a^m)\times\cdots\times D(a^{\frac{1}{2}})$, *there exists a unique solution* u *of* (1), *satisfying boundary conditions such that* $\left(u(t, x), \dfrac{\partial u}{\partial t}(t, x), \cdots, \dfrac{\partial^{2m}u}{\partial t^{2m}}(t, x)\in C^0([0, T], D(a^m)\times D(a^{m-\frac{1}{2}})\times\cdots\times D(a^{\frac{1}{2}})\times L^2(\Omega)\right)$. *Here* $D(a)=H^2(\Omega)\cap H^1_0(\Omega)$ *or* $\left\{u\in H^2(\Omega)\,\middle|\,\left(\dfrac{\partial}{\partial n}+\rho(x)\right)u\Big|_\Gamma=0\right\}$. *Furthermore* $\dfrac{\partial}{\partial n}$ *is the conormal derivative of* a *and* $\rho(x)\in C^\infty(\Gamma)$.

To prove the theorem above mentioned, we need to extend our singular integral operators defined on $\boldsymbol{R}^n_+$ to ones defined over Ω ([11]).