82. *On Classes of Summing Operators. I*

By V. CRAIU and V. ISTRĂŢESCU
Department of Mathematics, University of Bucharest and Institute of Mathematics, Rumanian Academy

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1. Let X and Y be Banach spaces and $L(X, Y)$ be the space of all bounded operators from X into Y with the norm

$$\|T\| = \sup_{\|x\|\le 1} \|Tx\|$$

Some important classes of operators in $L(X, Y)$ were considered in connection with the classes of nuclear operators and Hilbert-Schmidt operators; a unified theory of these is the theory of p-absolutely summing operators of A. Pietsch [4].

It is the aim of the present note to give a generalization of the class of A. Pietsch. In the first step our generalization has its origin in the generalization of L_p-spaces given by Orlicz.

In a paper which will follow we give a new generalization inspired from theory of modulared spaces [3].

2. Complementary functions in the sense of Young.

For $t\geqslant 0$ let $y=\varphi(t)$ be a non-decreasing function such that $\varphi(0)=0$, φ does not vanish identically and φ is left continuous for $t>0$; let ψ be the left continuous inverse of φ. Define the function

$$\phi(t)=\int_0^t \varphi(s)ds \qquad \psi(s)=\int_0^s \psi(r)dr$$

for $t, s\geqslant 0$. The pair (ϕ, ψ) is called complementary Young functions; a basic result is

$$ts \leqslant \phi(t)+\psi(s)$$

and equality holds if and only if $s=\varphi(t)$, $t=\psi(s)$.

3. ϕ-absolutely summing operators.

Let X, Y be Banach spaces and $T\in L(X, Y)$.

Definition 1. For $T\in L(X, Y)$ and (ϕ, ψ) be complementary Young functions we define the number $a_\phi(T)$ us

$$a_\phi(T)=\inf\left\{c, \sum_1^n \phi(\|Tx_i\|)\right\} \leqslant \phi(c) \sup_{\|x^*\|\le 1}\left(\sum_1^n \phi(|x^*(x_i)|)\right)$$
$$x_i\in X,\ i=1, 2, \cdots, n.$$

If $a_\phi(T)<\infty$ we call T, ϕ-absolutely summing.

Remark. Since $\phi(t)=\dfrac{1}{p}t^p$, $\psi(s)=\dfrac{1}{q}s^q$ is a pair of complementary functions, for this functions we obtain the class of A. Pietsch.