77. On the Structure of Certain C*-Algebras

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(Comm. by Kunihiko KODAIRA, M. J. A., May 12, 1969)

A C*-algebra A is said to be *elementary* if A is isomorphic to the C*-algebra $LC(H)$ of the totality of compact operators on a Hilbert space H. The dual $\hat{A}$ of any elementary C*-algebra A consists of a single element (cf. 4.1.5 in [1]), conversely any separable C*-algebra is elementary if its dual consists of a single element [5], where the dual $\hat{A}$ is the set of all unitary equivalence classes of irreducible representations* of A. A C*-algebra A is called a CCR-algebra if $\pi(A)\subset LC(H_\pi)$ and a GCR-algebra if $\pi(A)\cap LC(H_\pi)\neq(0)$ (cf. [6]), for all irreducible representations π of A, where H_π denotes the representation space of π.

In this paper, we present some results on the structure of GCR-algebras whose dual consists of a finite number of elements.

Lemma 1. *If A is a separable* C*-*algebra and Card $\hat{A}\leqq\aleph_0$, then A is of type I.*

Proof. Let π be an irreducible representation of A. If $\pi(A)\cap LC(H_\pi)=(0)$, then, by [2], there is a family of mutually inequivalent irreducible representations of A which has the cardinal number of continuum. This fact is contrary to our assumption. Therefore we have $\pi(A)\cap LC(H_\pi)\neq(0)$ and, by [4] and [6], A is a C*-algebra of type I.

Lemma 2. *Let $(A_i)_{i\in I}$ be a family of non-zero* C*-*algebras and let A be the product* C*-*algebra of A_i's. Then we have*

$$\hat{A}=\bigcup_{i\in I}\{\rho_\pi\mid\pi\in\hat{A}_i\} \tag{1}$$

if and only if the index set I is a finite set, where ρ_π is a representation $(x_i)_{i\in I}\to\rho_\pi((x_i)_{i\in I})=\pi(x_i)$ of A.

Proof. Suppose that (1) is satisfied. Let B be the restricted product C*-algebra of A_i's (cf. 1.9.14 in [1]). Then B is a closed two-sided ideal of A. Assume that $B\subsetneqq A$, then there is an irreducible representation ν such that $\nu(B)=(0)$. By the assumption, there is an irreducible representation π of A_i, such that $\nu=\rho_\pi$. Then we have $(0)=\nu(B)=\rho_\pi(B)=\pi(A_i)=\rho_\pi(A)=\nu(A)$. This is a contradiction. Therefore $B=A$. Since A_i is non zero, I is a finite set.

Conversely, let I be a finite set. Each A_i is a closed two-sided ideal of A. Let π be an irreducible representation of A. Since I is a finite set, there is an index $i\in I$ such that $\pi(A_i)\neq(0)$. Let $j\in I$ be any

*) Throughout this paper, we mean by an irreducible representation a non-trivial one.