76. *A Class of Purely Discontinuous Markov Processes with Interactions.* I[1]

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1. Starting with Kac's model of Boltzmann equation,[2] McKean [5]-[7] introduced an interesting class of Markov processes with non-linear generators. These processes describe the motion of one particle under the interactions between infinite number of similar particles.[3]

We construct a class of these processes by modifying the classical method of Feller [1]. The forward equation with possibly unbounded and temporally inhomogeneous equation is considered. Interactions can be infinitely multifold.

I thank S. Tanaka and H. Tanaka who sent me the manuscript of [9] and a part of [8], respectively, before publication.

2. First, we consider the simpliest model with binary interactions. Let R be a locally compact space with countable bases and let $B(R)$ be the topological Borel field. The forward equation is

$$(1)\qquad \frac{d}{dt}P^{(f)}(s,x,t,E)=\int_R P^{(f)}(s,x,t,dy)A^{(P^{(f)}_{s,t})}(t,y,E),$$

$$-\infty\leq t_0\leq s<t\leq t_1\leq+\infty$$

$$P^{(f)}(s,x,t,E)\to\delta_x(E),\quad \text{as } t\to s,$$

where initial distribution f at time s and the solution $P^{(f)}(s,x,t,E)$ are substochastic measures, and

$$P^{(f)}_{s,t}(E)=\int_R f(dx)P^{(f)}(s,x,t,E).$$

Kernel $A^{(u)}$ indexed by a substochastic measure u, is

$$(2)\qquad A^{(u)}(t,x,E)=\int_R u(dx_1)q(x_1\,|\,t,x)(\pi^0(x_1\,|\,t,x,E)-\delta_x(E)),$$

where $q(x_1\,|\,t,x)$ is non-negative and majorized by another function $q(t,x)$ which is *bounded on any compact* (t,x)*-set*. $\pi^0(x_1\,|\,t,x,E)$ is a *probability measure with no mass at point* x. $q(t,x)$, $q(x_1\,|\,t,x)$ and $\pi^0(x_1\,|\,t,x,E)$ are measurable in (t,x) and (x_1,t,x), and continuous in t when other variables are fixed. Intuitively, $\pi^0(x_1\,|\,t,x,E)$ indicates the

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2) Introduced by Kac [4] related with a justification of Boltzmann equation.

3) This explanation is justified by the "propagation of chaos" proposed by Kac. The reader can consult Kac [4] and McKean [5, 6].