75. *Absolute Convergence of Fourier Series*

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1. Introduction and theorems.

1.1. Let f be an even integrable function, with period 2π and its Fourier series be

(1) $$f(x)\sim\sum_{n=1}^{\infty}a_n\cos nx.$$

R. Mohanty [1] has proved the following

Theorem I. *If* $(I,1)$ *the function* $\log(2\pi/t)f(t)$ *is of bounded variation on the interval* $(0,\pi)$ *and* $(I,2)$ *the sequence* $(n^\delta a_n)$ *is of bounded variation for a* $\delta>0$, *then* $\sum|a_n|<\infty$.

Later one of us [2] proved

Theorem II. *If* $(II,1)$ f *is of bounded variation and* $\int_0^\pi\log(2\pi/t)\cdot|df(t)|<\infty$ *and* $(II,2)$ *the sequence* $(n^\delta\Delta(na_n))$ *is of bounded variation for a* $\delta>0$, *then* $\sum|a_n|<\infty$.

Recently R.M. Mazhar [3] has proved

Theorem III. *If the condition* $(II,1)$ *is satisfied and* $(III,2)$ *the sequence*

$$e^{-n^\alpha}\sum_{m=1}^{n}e^{m^\alpha}a_m\qquad(n=1,2,\cdots)$$

is of bounded variation for an $\alpha,0<\alpha<1$, *then* $\sum|a_n|<\infty$.

The conditions $(I,1)$ and $(II,1)$ are mutually exclusive and $(I,2)$ and $(II,2)$ are also. The condition $(III,2)$ is weaker than $(II,2)$ ([3], Lemma 2) and then Theorem III is a generalization of Theorem II.

1.2. Our object of this paper is partly to prove Theorem I without using Tauberian theorem and partly to generalize the condition $(I,1)$as Theorem III, namely:

Theorem 1. *Suppose that the sequence* (m_k) *is positive and increasing and satisfies the following conditions*:

(2) $$m_{k+1}/m_k\leqq A,\ M_k/m_k\leqq Ak^{\delta-\varepsilon}\qquad\text{for an }\varepsilon,0<\varepsilon<\delta<1,$$

where $M_k=m_1+m_2+\cdots+m_k$ *and there is an integer* p *such that*

(3) $$|\Delta^{p-1}(M_j\Delta(1/m_j))|\leqq A/j\qquad\text{for all }j>1.$$

If $(1,1)$ f *is of bounded variation and* $\int_0^\pi\log(2\pi/t)|df(t)|<\infty$ *and* $(1,2)$ *the sequence*