104. *Convergence of Transport Process to Diffusion*

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Let us consider a transported particle in the transprot medium G, which is bounded or unbounded domain of R^n. Suppose that it travels in a straight line and interacts with the medium with probability $k\Delta + o(\Delta)$ during time t and $t+\Delta$. The scattering distribution of velocity from $c\omega$ to $c\omega'$, $\omega' \in d\omega'$ at point $x \in G$ is assumed given by $\pi_x(d\omega')$. If the particle hits the boundary of G, then it dies. Under these assumptions, the position $X(t)$ and velocity $V(t)$ of the particle at time t together make up a Markov process $(X(t), V(t))$.

The purpose of this paper is to show that when $c\to\infty$, the process $X(t)$ converges to a diffusion under some additional assumptions (Assumptions I, II, and III).

The same result has been obtained in case of one-dimensional transport process by N. Ikeda, H. Nomoto [1] and M. Pinsky [3]; in case of two-dimensional isotropic one by A. S. Monin [2] and T. Watanabe [6]; in case of multi-dimensional isotropic one by S. Watanabe and T. Watanabe [5].

1. Let G be bounded or unbounded domain of n-dimensional Euclidian space R^n. Suppose that the boundary ∂G of G is smooth, if it exists. Let Ω be a bounded set in R^n. Let denote by S the product space of R^n and Ω, and by $C_0(S)$ the Banach space of bounded continuous function on S vanishing at infinity and at boundary point (x, ω) such that $(\boldsymbol{n}_x, \omega)\leq 0$, where $\boldsymbol{n}_x$ is an inner normal vector at $x \in \partial G$. Let T_t^c, $t\geq 0$, be the strongly continuous positive contraction semigroup on $C_0(S)$ with infinitesimal generator A^c given by:

$$A^c f(x, \omega) = c(\omega, \operatorname{grad} f) + k\int_\Omega [f(x, \upsilon) - f(x, \omega)]d\pi_x(\upsilon),$$

where $(\omega, \operatorname{grad} f) = \sum_{i=1}^n \omega_i \frac{\partial}{\partial x_i} f$, $\omega = (\omega_1, \cdots, \omega_n)$, and π_x $(x \in R^n)$ is a probability measure on Ω. We call this semigroup T_t^c, $t\geq 0$, *the transport process with speed c.*

Now let $C_0(G)$ be the Banach space of bounded continuous function on G vanishing near the boundary ∂G and at infinity, and $C_{\mathcal{K}}^3(G)$ be the subspace of $C_0(G)$ of function with compact support, whose thrice derivatives belong to $C_0(G)$. Let T_t^D, $t\geq 0$, be the strongly continuous positive contraction semigroup on $C_0(G)$ of diffusion determined by: