103. On the Global Solution of a Certain Nonlinear Partial Differential Equation

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1. Introduction. We consider the following fourth order partial differential equation

$$(1)\qquad \partial^2 y/\partial t^2=(1+\alpha(\partial y/\partial x)^{2p})\partial^2 y/\partial x^2-\beta\partial^4 y/\partial x^4,$$

where α and β are positive constants and $p=1,2,\cdots$, which is deeply connected with the study of the anharmonic lattice (see [1]).

Here we consider the initial-boundary value problem for (1) with initial values

$$(2)\qquad y(0,x)=f(x),\qquad \partial y/\partial t(0,x)=g(x),$$

and with periodic boundary condition

$$(3)\qquad y(t,x)=y(t,x+1)\qquad \text{for all } x \text{ and } t.$$

Then we have the following theorem being concerned with the global solution for the problem:

Theorem. *For every $\alpha>0$, $\beta>0$, and for every real 1-periodic initial functions $f\in W_2^{(6)}(0,1)$, $g\in W_2^{(4)}(0,1)$, there exists the unique function which satisfies* (1), (2) *and* (3) *in the classical sense in the whole (t,x) plane.*

The method of proof is the semi-discrete approximation similar to that presented by Sjöberg [2].

The authors were announced by Nisida [3] that he independently treated the same problem by means of the theory of semi-groups.

2. Proof of existence of the global solution. In order to prove the existence of the desired solution we employ the following semi-discrete approximation:

$$(4)\qquad \begin{aligned} &d^2y_N(t,x_r)/dt^2=D_+[D_-y_N(t,x_r)+\alpha(D_-y_N(t,x_r))^{2p+1}/2p+1]\\ &\qquad -\beta D_+^2D_-^2y_N(t,x_r),\qquad r=1,2,\cdots,N\\ &y_N(0,x_r)=f(x_r),\qquad dy_N/dt(0,x_r)=g(x_r),\qquad r=1,2,\cdots,N,\\ &y_N(t,x_r)=y_N(t,x_{r+N}),\qquad r=1,2,\cdots,N\qquad \text{and all } t\end{aligned}$$

where the mesh-width $h=1/N$, N natural number, $x_r=rh$ and the difference operators D_+ and D_- are defined by

$$hD_+y(x_r)=y(x_{r+1})-y(x_r),\qquad hD_-y(x_r)=y(x_r)-y(x_{r-1}).$$

For every $h>0$ the solution of the problem (4) uniquely exists on the basis of the theory of ordinary differential equations. The solution $y_N(t,x_r)$, fixed N, is a grid-function defined for $x_r=rh$. We may write $y_N(t,x_r)=y_r(t)$ for the sake of simplicity.