102. *Some Properties of Porges' Functions*

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§ 1. **Introduction.** For fixed g and $s \in Z$,[1] let $f(n)$ be the sum of the sth powers of the digits in the scale of g of the natural number n. Porges [1], Isaacs [2], and Stewart [3] studied the properties of this function $f(n)$. The sequence $\{f^k(n)\}_{k=0}^{\infty}$, where $f^0(n)=n$, and $f^k(n) =f\{f^{k-1}(n)\}(k \in Z)$, is periodic for every $n \in Z$ (see [4]). K. Iséki [5], [6] reported all the periods for $s=3, 4, 5$, when $g=10$. Integers X and Y are said to be f-related if and only if there are non-negative integers l and m such that $f^l(X)=f^m(Y)$. Being f-related is an equivalence relation dividing Z into N disjoint sets of f-related integers (see [2]). Now let $P(g)$ be the set of all the periods of the sequences $\{f^k(n)\}_{k=0}^{\infty}(n \in Z)$ and let $M(g)$ be $\max\{\bar{A} \mid A \in P(g)\}$, where $\bar{A}$ is the number of elements of A when $s=2$. Then in the case of $s=2$, $N=N(g)$ is obviously the number of the elements of $P(g)$. In § 2 we will prove the following

Theorem 1. $\overline{\lim_{g\to\infty}} M(g)=\infty$ (1),

and

Theorem 2. $\overline{\lim_{g\to\infty}} N(g)=\infty$ (2).

When the circulation of $\{f^k(n)\}_{k=0}^{\infty}$ begins at $k=k(n)$th term, we get the sequence $\{h(n)\}_{n=1}^{\infty}$, where $h(n)=f^{k(n)}(n)$. In the case of $(g, s)=(3, 2)$, as easily proved, $H=\{h(n) \mid n \in Z\}=\{1, 2, 4, 5, 8\}$. In § 3, we will prove the following

Theorem 3. *For every pair (a, l), where $a \in H$, $l \in Z$, there exist infinitely many natural numbers k such that $h(k)=h(k+2)=\cdots=h(k+2l-2)=a$,*

Theorem 4. *Let $1 \leqq l \leqq 5$. For a given repeated permutation $E=(\xi_1, \xi_2, \cdots, \xi_l)$, where $\xi_\nu=1$ or 5, there exist infinitely many numbers b such that $(h(b), h(b+2), \cdots, h(b+2l-2))=(\xi_1, \xi_2, \cdots, \xi_l)$,*

Theorem 5. $(h(c), h(c+2), \cdots, h(c+10)) \neq (1, 5, 1, 1, 5, 1)$ *for all $c \in Z$ and*

Theorem 6. *Let $T(l)$ denote the number of the repeated permutations $(\xi_1, \xi_2, \cdots, \xi_l)$, where $\xi_\nu=1$ or 5, which can be realized by infinitely many number of finite partial sequences consist of l consecutive terms of $\{h(2n-1)\}_{n=1}^{\infty}$, then*

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1) Z is the set of all natural numbers.