101. *Paracompactness of Product Spaces*

By Jingoro SUZUKI
Department of Mathematics, Faculty of Education, Chiba University

(Comm. by Kinjirô KUNUGI, M. J. A., June 10, 1969)

§1. As for the normality of the product of a topological space X with a paracompact space Y, several sufficient conditions have been obtained. Indeed, as far as I know, the following are all the cases for which the product $X \times Y$ has been proved to be paracompact for any paracompact space Y.

(a) X is a paracompact space which is a countable union of locally compact closed subsets (K. Morita [6]).[1]

(b) X is the image under a closed continuous mapping of a locally compact, paracompact, Hausdorff space (T. Ishii [1]).

(c) X is a regular space which has two coverings $\{C_\gamma \mid \gamma \in \Gamma\}$ and $\{U_\gamma \mid \gamma \in \Gamma\}$ such that

(i) C_γ is compact, U_γ is open and $C_\gamma \subset U_\gamma$ for each $\gamma \in \Gamma$, and

(ii) $\{U_\gamma \mid \gamma \in \Gamma\}$ is order locally finite[2] (Y. Katuta [2]).

All spaces considered in this paper are assumed to be Hausdorff. Ishii has showed that his result (b) is not covered by Morita's result (a). On the other hand, Katuta has showed that his result (c) covers Morita's result (a) (in the case when X and Y are regular spaces) and that his result is contained neither in Morita's result nor in Ishii's. Moreover, Katuta stated that he did not know whether his result covers Ishii's result (cf. [2], p. 616).

In this note we shall answer this question negatively in §2. In §3 we shall give a sufficient condition for X to possess the property that the product space $X \times Y$ be paracompact for any paracompact space Y, such that all the conditions mentioned above are obtained as special cases of our condition.

§2. Let M be a compact Hausdorff space such that $M-\{m_0\}$ is not paracompact for some distinguished point m_0. Let $\{M_\alpha \mid \alpha \in A\}$ be an uncountable collection of copies of M. We denote by P the topological sum of $\{M_\alpha\}$. Let X be the quotient space of P which is obtained by identifying all copies of m_0, and let f be the natural mapping

1) Morita has assumed that X and Y are paracompact normal spaces instead of paracompact Hausdorff spaces.

2) A collection $\{A_\lambda \mid \lambda \in \wedge\}$ of subsets of a topological space is called order locally finite, if we can introduce a total order $<$ in the index set $\wedge$ such that for each $\lambda \in \wedge$ $\{A_\mu \mid \mu < \lambda\}$ is locally finite at each point of A_λ.